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XIII. *Notes on the Plug Permeameter.*

By CHARLES V. DRYSDALE, *D.Sc.**

[Plates VII. & VIII.]

IN 1901 the writer described a permeameter†, devised with the object of enabling permeability-tests to be made on castings and forgings for dynamo-work. The apparatus consisted of a special drill, which could be employed to bore a hole in the casting or forging, leaving, however, a small piece of the metal in the form of a rod or pin standing in the centre of the hole. A split iron plug, arranged to fit both the conical sides of the hole and over the pin, was employed to complete the magnetic circuit; and this plug carried a bobbin wound with magnetizing and search coils. A special portable testing set was also designed with the object of making permeability-tests by direct reading of H and B.

It was noticed at the time that the magnetization-curves obtained by this method fell below those obtained by measurements on a long bar of the same material. This would naturally be attributed to a bad magnetic joint between the pin and plug; but experiments made at the time seemed to point rather to the effect being due to the reluctance of the return path at the point of entrance of the flux, due to the concentration of the lines of force at these points. This seemed probable from the fact that the readings were remarkably consistent when the plug was removed and replaced.

This hypothesis rendered it probable that there might be a more or less definite relation between the value of the true magnetizing-force H in the specimen, and the apparent magnetizing-force $H' = \frac{4\pi}{10}\frac{Cn}{l}$; and arrangements were made to test this relation. The most perfect method was suggested by the double-yoke method of Prof. Ewing. It consisted in finding the relation of the induction density and the magnetizing current for the ordinary drilled specimen, and afterwards when the length of the specimen was doubled by drilling

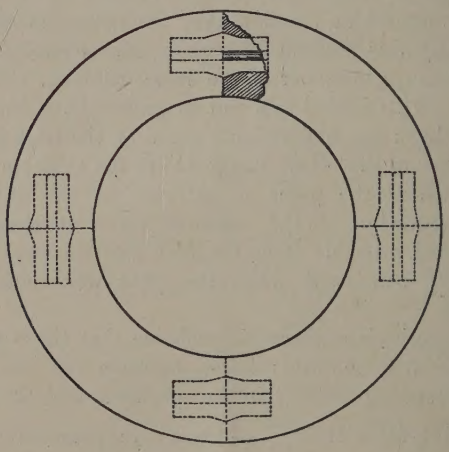
* Read March 27, 1908.

† Proc. Inst. Electr. Eng. vol. xxxi. p. 283.

to a greater depth with a second drill. The increase of the magnetizing current for the same induction should give the true H. A large number of tests were made in this manner, but proved of little value, as, although the first drilling presented no difficulties, it was found impossible to perform the second drilling accurately enough to preserve parallelism of the pin.

Recourse was therefore had to the more simple device of obtaining some specimens of cast and wrought iron and steel, in the form of rings; testing them by the ordinary ring method, and afterwards cutting them up and drilling, for tests by the plug. Rings of this kind were made for us by Messrs. G. Wailes of Euston Road, and were of ordinary cast iron (O.C.I.), special cast iron (S.C.I.), wrought iron (W.I.), and mild steel (M.S.). Two rings of each material

Fig. 1.



were tested, and each ring was cut into four quadrants. Each of these quadrants was drilled in both ends, so that eight drillings were made in each ring. Readings of B were then made with the plug method, at nominal values of H of 30 and 100 on the eight drillings, and the drilling selected for final test, the readings on which corresponded to the mean of the eight. It may be mentioned that the whole of the

64 drillings on the eight rings were made with the same drill, which was none the worse after. None of the drillings gave a bad fit with the plug. Fig. 1 shows the form and mode of cutting and drilling the ring, which was 5 in. external diameter and 1 in. square in section ; while Table I. shows an example of the readings on the various drillings of one ring, and the amount of variation usually found. It should be specially noted that these variations are not apparently due to variation in the fit of the plug, but rather to actual differences in the magnetic quality of the iron.

TABLE I.

Readings on Wrought Iron Ring.

Hole No.	Fit.	H=30. B.	H=100. B.
1	Fair.	13,140	15,600
2	Moderate.	12,880	15,820
3	Good.	12,400	15,130
4	Good.	12,040	14,550
5	Fair.	11,730	14,550
6	Good.	12,930	15,080
7	Good.	11,700	14,190
8	Fair.	11,670	14,550
1 (Check-reading)		.....	15,600
Mean		12,250	14,900

Figs. 2–9, Pl. VII., show magnetization-curves taken on the ring and by the plug on the mean drilling for each specimen. It will be noted that the curve obtained by the plug lies below that for the ring in every case ; but there is no evidence to show that this is due to an air-gap. If such a gap were present, of approximately equal amount in each of the specimens, the shearing-over of the curves of the wrought-iron and mild-steel specimens would be much more marked in comparison to those of the cast-iron specimens, owing to the much greater permeability and induction-density in the former.

Table II. is obtained from the curves by finding the value of H′, the nominal magnetizing force in the case of the plug-test, for each value of H from the ring-test, for a given induction-density; and fig. 10, Pl. VIII., gives curves showing

the relation of H′ to H for each of the materials and for the mean of all. It will be observed that although there are considerable differences between the values in the various columns, they appear to have no relation to the permeability of the specimen, and must therefore be regarded as accidental. In fact, the difference between H and H′ is greater in the case of the special cast-iron than for either the wrought-iron or mild-steel specimens, which is exactly contrary to the effect which would be produced by an air-gap. The difference must consequently be attributed to the shortness of the specimen; and the results therefore may have some general value in indicating the amount of the end effect in the case of yoke-permeameters, in which the specimen is only five diameters long.

TABLE II.

Relation of H to H′.

H.	Mean H′ for O.C.I.	Mean H′ for S.C.I.	Mean H′ for W.I.	Mean H′ for M.S.	Mean.
0	0	0	0	0	0
5	8·5	9·7	8 5	8·0	8·7
10	17·2	19·2	18·75	18·5	18·4
15	26·2	29·4	28 2	29·0	28·2
20	34·0	39·0	35·7	37·7	36·6
25	42·2	47·7	42·2	46·5	44·7
30	49·5	56·0	48 0	54·2	51·5
35	57·0	65·25	54·0	61·0	59·3
40	63·5	73·75	60·5	66·5	66·0
45	69·5	81·75	64·5	72·2	71·9
50	76·5	90·75	69·7	77·5	78·5
55	83·5	98·25	75·0	82·0	84·7
60	90 0	106·0	79·5	88·5	91·0
65	96·5	114·0	83·7	93·2	96·9
70	103·0	121·5	89·0	97·5	102·8
75	109·2	129·0	94·2	102 2	108·6
80	115·0	135·7	99·0	108·5	114·6

In order to show the result of correcting the indications of the instrument, the curves figs. 2–9 have points marked by crosses, in which the readings of the plug-permeameter are shifted backwards along the H axis by amounts corresponding to the mean curve in fig. 10 (Pl. VIII.). The agreement with the ring-tests then becomes fairly good. Fig. 11, Pl. VIII., shows readings taken by one of the portable testing sets, in which the H scale has been marked off in this way. The

curves are those given by the ring method, while the points are taken by the testing set. The agreement is as good as in most of the recognized permeameters.

Fig. 12, Pl. VIII., shows two curves taken by the plug permeameter: one on a cast-steel pole-piece for a large generator, which was sent us by the dynamo-builders under the impression that it was of poor magnetic quality. The test shows the permeability to be very good. The other curve is for another specimen of steel, and indicates the remarkable variations of quality which are found in practice.

It should be mentioned that when the plug used in these tests was replaced by a second one, the results agreed within an accuracy of $1\frac{1}{2}$ per cent.

The general conclusion to be drawn from these experiments is that the plug permeameter, in reasonably careful hands, gives as good results as any other method of iron testing, so far as permeability tests are concerned. In a recent paper by Mr. Murdoch* this permeameter was criticized as employing a very small specimen; but this is precisely what has been aimed at in its design. It appears to be of great importance that permeameters for castings or forgings should be capable of being employed on the bulk of the metal itself, without extra machined specimens, and up to the present no other instrument has been devised which meets this requirement.

The writer's hearty thanks are due to Mr. A. C. Jolley for the considerable amount of work involved in carrying out the tests. Mr. C. M. Dowse has also rendered considerable assistance in the later tests.

Discussion.

Prof. S. P. Thompson said the Author's instrument commended itself for its usefulness and ingenuity. He would like to know how far its results were comparable with those obtained from ring and double yoke methods. He was not sure that the Author's explanation that the discrepancies were due to end effects was the correct one. In drilling the hole in the casting the magnetic properties of the iron were altered and the hysteresis increased.

Mr A. Campbell remarked that from his experience of several

* Electrician, vol. lx. p. 245.

varieties of permeameters he was not at all surprised to find that Dr Drysdale had had to calibrate his permeameter by the purely experimental method of comparison with the trustworthy ring method. By averaging the different calibrations given by various types of material he appeared to be able to obtain fairly good results, but there were other permeameters which gave as satisfactory results without any empirical calibration.

Dr A. Russell said he appreciated the ingenuity displayed in the Drysdale permeameter. He suggested the following as the probable cause of part of the discrepancy between the magnetization curves got by the ring method and by the permeameter. In the former method the mean flux and the mean magnetizing force over the cross section of the ring were measured. The ring experimented on was of appreciable radial depth, so that the magnetic force on the inner circumference of the ring was about 50 per cent. greater than that on the outer circumference. The permeability of the iron at the greater force would probably be very different from that at the smaller force. The mean value, therefore, of the flux density over the cross sectional area of the ring might possibly be something quite different from the value it would have if the magnetic force were constant and equal to the mean magnetic force. The errors due to this cause in the ring method, even when the ring was narrow, were sometimes very appreciable. The criticism also applied to many cases in which electricians apply what they call the fundamental magnetic equation.

Mr W. Duddell asked if it was assumed that all the pins had the same diameter.

Dr C. V. Drysdale, in reply to Prof. Thompson, said that the end effect was not of the same nature as an air-gap or it would have had a more marked influence at high permeabilities. When the instrument was first introduced, a few tests had been made by drilling a specimen and testing it, and afterwards annealing and re-testing. No change had been observed, but possibly a greater number of tests would have shown variations. The size of the pin seemed not to vary by more than a mil whatever metal was drilled. As to Mr Campbell's remarks, he quite agreed that other permeameters were equally or more accurate, but this was the only one that could be used on the actual forging or casting, and it was surprising and gratifying to find that it was as accurate as it seemed to be. The thickness of the ring mentioned by Dr Russell was an important point. It was necessary in order to permit of drilling, but he was inclined to think that there would be little error from this cause except at low inductions.

XIV. *The Use of Shunts and Transformers with Alternate Current Measuring Instruments.* *By* CHARLES V. DRYSDALE, *D.Sc.**

[Plate IX.]

THE growing demand for instruments for alternate-current measurement, of considerable range and accuracy, has called attention to some of the difficulties attending their production, especially to their limited range. As alternate-current instruments measure the root-mean-square values of the P.D. or current, it necessarily follows that the deflecting torque falls very rapidly as the P.D. or current is decreased, For most commercial instruments, it may be taken that a ten-fold range of torque is the utmost possible for accurate reading, so that when the deflexion is proportional to the torque the range of an A.C. instrument can only be of the order of 3 to 1 except by the employment of double coils or auxiliary devices. The auxiliary devices which may be employed are transformers for electromagnetic ammeters, voltmeters, and wattmeters; shunts for ammeters and wattmeters, series resistances for electromagnetic voltmeters, and condensers and split resistances for electromagnetic voltmeters. Of these, the use of shunts and transformers may give rise to serious errors, and it is proposed here to investigate the amount of these errors and the conditions for their elimination.

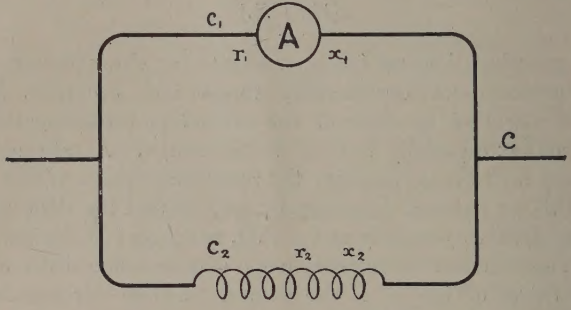
In dealing with any shunting or transforming device we must bear in mind that it may cause errors in two ways:—(*a*) by the multiplying power of the shunt or the ratio of the transformer being affected by frequency, &c.; and (*b*) by a phase displacement being introduced between the current or P.D. on the instrument and the main current or voltage. Ammeters and voltmeters are of course only affected by the first error, but the phase-displacement may be of much greater importance in wattmeters.

* Read March 27, 1908.

Shunts.

Multiplying Power.—In fig. 1, we have an instrument of resistance r_1 and reactance x_1 shunted by a circuit of resistance r_2 and reactance x_2. Then, if V is the P.D. between

Fig. 1.



the terminals, and C_1 and C_2 are the currents in the instrument and shunt respectively, we have

$$C_1 = \frac{V}{I_1} = \frac{V}{r_1 - jx_1} \quad \text{and} \quad C_2 = \frac{V}{I_2} = \frac{V}{r_2 - jx_2}.$$

Hence the total current in the circuit $C = C_1 + C_2$

$$= V\left\{\frac{1}{r_1 - jx_1} + \frac{1}{r_2 - jx_2}\right\} = V\frac{r_1 + r_2 - j(x_1 + x_2)}{(r_1 - jx_1)(r_2 - jx_2)}, \quad . \quad (1)$$

and the virtual current $\overline{C} = \overline{V}\frac{I}{I_1 I_2}$, where I is the impedance of the whole circuit formed by the instrument and shunt *in series*, and I_1 and I_2 are the impedances of the instrument and shunt respectively.

Consequently, since $\overline{C_1} = \frac{V}{\overline{I_1}}$ the multiplying power of the shunt

$$M' = \frac{\overline{C}}{\overline{C_1}} = \frac{I}{\overline{I_2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$M' = \sqrt{\frac{(r_1 + r_2)^2 + (x_1 + x_2)^2}{r_2^2 + x_2^2}} = \sqrt{\frac{\left(\frac{r_1 + r_2}{r_2}\right)^2 + \left(\frac{x_1 + x_2}{r_2}\right)^2}{1 + \left(\frac{x_2}{r_2}\right)^2}}$$

But the multiplying power of the shunt for direct currents $M = \frac{r_1 + r_2}{r_2}$ from which $r_1 = (M-1)r_2$. In addition $\frac{x_1}{r_1} = T_1 p$, where T_1 is the time constant of the instrument and p is 2π times the frequency as usual. Similarly $\frac{x_2}{r_2} = T_2 p$, where T_2 is the time constant of the shunt.

Putting these values in the expression above we have

$$M' = \sqrt{\frac{M^2 + \{(M-1)T_1 + T_2\}^2 p^2}{1 + T_2^2 p^2}},$$

which after a little further simplification reduces to

$$M' = M\sqrt{1 + \frac{M-1}{M^2} \frac{(M-1)T_1 + (M+1)T_2}{1 + T_2^2 p^2} (T_1 - T_2) p^2}. \quad (3)$$

This formula at once shows that if $T_1 = T_2$ the shunt has the same multiplying power for both direct and alternate currents, as is well known. It further shows that if

$$(M-1)T_1 + (M+1)T_2 = 0 \quad \text{or} \quad \frac{T_2}{T_1} = -\frac{M-1}{M+1}$$

the shunt is again correct. This would be the case if the instrument is shunted with a resistance r_2 and capacity K such that $K r_2 = \frac{M-1}{M+1} T_1$*.

If the instrument is inductive, and is shunted with a non-inductive resistance, $T_2 = 0$ and the formula reduces to

$$M' = M\sqrt{1 + \left(\frac{M-1}{M}\right)^2 T_1^2 p^2}. \quad . \quad . \quad . \quad (4)$$

As an example, a Kelvin centiampere balance was found on test to have a resistance of 62·4 ohms and an inductance (by secohmmeter) of 161 millihenrys. The time constant of this instrument was therefore ·00258 second. The instrument was then shunted with a non-inductive resistance, and it was found that a current of ·878 amp. gave a reading of ·17 amp. on the balance with direct current, thus giving a multiplying

* Mr. Alexander Russell has cast doubts on this formula, and I have found that it is only true in special cases. It was based on the assumption that a shunted condenser could be treated as an impedance with negative time constant, which is not strictly true.

power M of 5·16. Hence at 50 periods, which is the usual frequency of the supply, M′ should be

$$5{\cdot}16\sqrt{1+\frac{4{\cdot}16^2\times{\cdot}00258^2\times 314^2}{5{\cdot}16^2}}=6{\cdot}17;$$

while on testing a current of ·927 amp. gave a reading of ·15 amp. or a multiplying power of 6·18. It should be noticed that the formula may be expressed in the form $T_1=\frac{\sqrt{M'^2-M^2}}{(M-1)p}$, which gives us a convenient method of testing the inductances of ammeters, &c.

For commercial work, therefore, we see that the only legitimate method of employing shunts is to make the time constants of the instruments and shunts either negligible or approximately equal. If T_1 and T_2 are nearly equal since $\sqrt{1+h}=1+\frac{h}{2}$ nearly when h is small,

$$M'=M\left\{1+\frac{M-1}{2M^2}\frac{(M-1)T_1+(M+1)T_2}{1+T_2^2p^2}(T_1-T_2)p^2\right\}. \quad (5)$$

It will be noticed that for a given difference in the time constants the correcting term is greater the greater the value of M and the less that of T_2. Consequently, if we take in our formula M so large that M, (M+1) and (M−1) may be regarded as equal and at the same time neglect T_2p in comparison with unity, we have:

$$M'=M\left\{1+\frac{T_1+T_2}{2}(T_1-T_2)p^2\right\}=M\{1+(T_1-T_2)Tp^2\}, \quad (6)$$

where T is the average value of the two time constants T_1 and T_2.

This may be written in the form $\frac{\Delta M}{M}=T\,.\,\Delta T\,.\,p^2$, and finally

$$\frac{\Delta M}{M}=T^2p^2\frac{\Delta T}{T}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Consequently, to ensure that the ratio M shall not be altered by more than x per cent. with alternating current of given frequency, the time constants must be adjusted to equality within $\frac{x}{T^2p^2}$ per cent.

In the case of the Kelvin centiampere balance before mentioned, $T = \cdot00258$ at a frequency of 50, therefore $Tp = 314 \times \cdot00258 = \cdot810$ and $T^2p^2 = \cdot655$. Consequently the time constant or the self-induction of the shunt must be adjusted to $\frac{1}{\cdot655} = 1\cdot53$ per cent. for an accuracy of 1 per cent. in the multiplying power. The simplest method of adjusting or checking the shunt is of course to test the ratio with both D.C. and A.C. or with A.C. of two different frequencies.

Phase Displacement.—Returning to formula (1) and rationalizing the denominator, we get

$$C = \frac{V}{I_1^2 I_2^2}\{(r_1+r_2)-j(x_1+x_2)\}(r_1+jx_1)(r_2+jx_2)$$

$$= \frac{V}{I_1^2 I_2^2}\{(r_1+r_2)(r_1r_2-x_1x_2)+(x_1+x_2)(r_1x_2+r_2x_1)$$

$$+j[(r_1+r_2)(r_1x_2+r_2x_1)-(x_1+x_2)(r_1r_2-x_1x_2)]\},$$

and consequently

$$\tan\phi = \frac{(r_1+r_2)(r_1x_2+r_2x_1)-(x_1+x_2)(r_1r_2-x_1x_2)}{(r_1+r_2)(r_1r_2-x_1x_2)+(x_1+x_2)(r_1x_2+r_2x_1)}$$

where ϕ is the angle of lag of the main current behind V.

Inserting the values of M, T_1, and T_2 as before, we get

$$\tan\phi = \frac{T_1+(M-1)T_2+\{(M-1)T_1+T_2\}T_1T_2p^2}{M+\{(M-1)T_1^2+T_2^2\}p^2}p.$$

Similarly $\tan\phi_1 = T_1p$ where ϕ_1 is the lag in the instrument. Hence

$$\tan(\phi_1-\phi) = \frac{\tan\phi_1-\tan\phi}{1+\tan\phi_1\tan\phi}$$

is the tangent of the angle of lag of the current in the instrument behind the main current. Putting in the values of $\tan\phi_1$ and $\tan\phi$ and simplifying, we have as the result

$$\tan\psi = \tan(\phi_1-\phi)$$

$$= (M-1)(T_1-T_2)p\,\frac{1+T_1^2p^2}{M+\{MT_1^2+(M-1)T_1T_2+T_2^2\}p^2+\{(M-1)T_1+T_2\}T_1^2T_2p^4}$$

$$= \frac{M-1}{M}(T_1-T_2)p\left\{1-\frac{(1+T_1^2p^2)\{(M-1)T_1+T_2\}T_2p^2}{M+\{MT_1^2+(M-1)T_1T_2+T_2^2\}p^2+\{(M-1)T_1+T_2\}T_1^2T_2p^4}\right\} \quad (8)$$

The phase-displacement is consequently zero for either $M=1$ or $T_1=T_2$ as is obvious, and also for $1+T_1^2p^2=0$, which is impossible. If the shunt is non-inductive $T_2=0$ and $\tan\psi=\frac{M-1}{M}T_1p$; while if the instrument is non-inductive and the shunt inductive,

$$\tan\psi=-\frac{M-1}{M+T_2^2p^2}T_2p.$$

Again, if both T_1p and T_2p are small compared with unity, the expression for $\tan\psi$ reduces to $\frac{M-1}{M}(T_1-T_2)p$. This of course means that the tangent of the phase-displacement between the instrument and mains is the fraction $\frac{M-1}{M}$ of that between the instrument and shunt, as is obvious geometrically. Finally, if T_1 and T_2 are nearly equal,

$$\tan\psi=\psi=\frac{M-1}{M(1+T^2p^2)}(T_1-T_2)p \quad . \quad . \quad . \quad (9)$$

where T is the mean time constant as before. When T is small this reduces to the last expression.

As an illustration of the error produced by shunting a wattmeter, we may assume a case where the time constant of the main circuit has a value of ·0025 as in the Kelvin balance above cited, and suppose that it is shunted with a non-inductive shunt of a nominal multiplying power of 10. We then have $M=10$, $T_2=0$, and T_1p at 50~

$$=314\times\cdot0025=\cdot785.$$

Hence

$$M'=M\sqrt{1+\left(\frac{M-1}{M}\right)^2T_1^2p^2}=10\sqrt{1+\cdot81\times\cdot785^2}=12\cdot25.$$

The phase-displacement is

$$\tan\psi_1=\frac{M-1}{M}T_1p=\frac{9}{10}\times\cdot785=\cdot705;$$

so that the current in the wattmeter lags about 35° behind that in the main circuit. Hence the instrument would read 22·5 per cent. too low owing to the error in the ratio, while the phase-displacement would cause the power factor to be apparently about ·5 when actually zero.

This strikingly illustrates the enormous errors which may be produced by shunts in wattmeters. For any given small displacement ψ we have from (9)

$$\frac{T_1 - T_2}{T} = \frac{\Delta T}{T} = \frac{M}{M-1}\left(\frac{1-T^2p^2}{Tp}\right)\psi. \quad . \quad . \quad (10)$$

and ψ will then be the maximum error in the power factor for loads nearly in quadrature. Consequently, in the above case, if the error in the power factor is not to exceed ·01,

$$\frac{\Delta T}{T} = \frac{10}{9}\frac{(1-\cdot 785^2)}{\cdot 785} \times \cdot 01 = \cdot 00542 \quad \text{or} \quad \cdot 54 \text{ per cent.};$$

and we saw before that for the ratio to be correct to 1 per cent.

$$\frac{\Delta T}{T} = 1\cdot 53 \text{ per cent.}$$

Transformers.

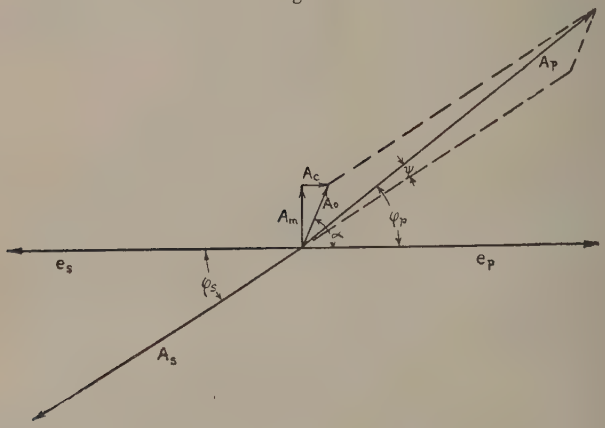
In dealing with the application of transformers to instruments, we shall have to consider separately their employment as "current transformers" or as "voltage transformers"; the former being employed in conjunction with ammeters for the purpose of utilizing instruments of convenient range, and to insulate the instrument from high voltage circuits; the latter with electromagnetic or hot-wire voltmeters to avoid undue waste of energy, and again as with ammeters to disconnect them from the high-pressure circuits. In wattmeters and energy-meters of the induction type, transformers are frequently employed in both the main and shunt circuits simultaneously.

The relations between the primary and secondary currents and voltages have been obtained by Steinmetz and others from the well-known vector diagram, but it will be well to treat the problem *ab initio* as we require the results to be exhibited in the most convenient form.

Current Transformers.—It is most convenient to deal at first with the magnitudes and phase relations of the currents, and we can afterwards apply the results to the investigation of the voltage relations.

Let A_p = primary ampere turns $= C_p n_p$.
A_s = secondary " " $= C n_s$.
A_m = magnetizing " " $= C_m n_p$.
A_c = core loss " " $= C_c n_p$.
A_0 = no load " " $= C_0 n_p$.
ϕ_p = lag of primary current behind core E.M.F.
ϕ_s = lag of secondary " " " "
α = lag of no load " " " "
ψ = phase displacement between primary and secondary currents.

Fig. 2.



Then we have

$$\hat{A}_p + \hat{A}_s = \hat{A}_0 \text{ vectorially;}$$

or writing the currents in the Steinmetz notation,

$$\hat{A}_s = -A_s \cos \phi_s - jA_s \sin \phi_s \quad . \quad . \quad . \quad . \quad (11)$$

$$\hat{A}_0 = A_c + jA_m.$$

Hence

$$\hat{A}_p = A_0 - A_s = A_s \cos \phi_s + A_c + (A_s \sin \phi_s + A_m) \quad . \quad (12)$$

and

$$\overline{A}_p = \sqrt{(A_s \cos \phi_s + A_c)^2 + (A_s \sin \phi_s + A_m)^2}$$

$$= \sqrt{A_s^2 + 2A_s(A_c \cos \phi_s + A_m \sin \phi_s) + A_0^2}.$$

If A_0 is small in comparison with A as it should always be in practice, we may write

$$\frac{A_p}{A_s} = 1 + \frac{A_c}{A_s}\cos\phi_s + \frac{A_m}{A_s}\sin\phi_s. \quad . \quad . \quad (13)$$

If R_c is the ratio of the primary to secondary currents, and R_t the ratio of secondary to primary turns, we have

$$R_c = R\left(1 + \frac{A_c}{A_s}\cos\phi_s + \frac{A_m}{A_s}\sin\phi_s\right). \quad . \quad . \quad (14)$$

Hence the ratio of the currents can only be equal to the transformation ratio if A_c and A_m are both zero or

$$\tan\phi_s = -\frac{A_c}{A_m} = -\cot\alpha.$$

This implies that the secondary current should be in quadrature with the no load current, as is geometrically obvious.

To test the constancy of transformation we have

$$\Delta R_c = R_t\left\{\Delta\left(\frac{A_c}{A_s}\right)\cos\phi_s + \Delta\left(\frac{A_m}{A_s}\right)\sin\phi_s\right\}, \quad . \quad (15)$$

which shows that for constant ratio of transformation both A_c and A_m must be proportional to A_s or that

$$\tan\phi_s = -\frac{\Delta\left(\frac{A_c}{A_s}\right)}{\Delta\left(\frac{A_m}{A_s}\right)}. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

For constant secondary impedance A_s is proportional to the core E.M.F. and hence to the induction density in the core, B. Hence the current vector should be perpendicular to the curve of which the ordinate is $\frac{A_m}{A_s}$, representing the reluctance of the circuit, and the abscissa is $\frac{A_c}{A_s}$ or the ratio of the core-loss current to B. It is obvious that this is when the secondary load is a leading one, and this result is contrary to the frequently quoted experiments of Mr. Campbell *, who

* Phil. Mag. vol. xlii. p. 271; Journal I. E. E. vol. xxxiii. p. 654.

is claimed to have found experimentally that better constancy of ratio is obtained with an inductive instrument. In the case, however, where there is an air-gap in the transformer, the curve connecting $\frac{A_m}{A_s}$ with $\frac{A_c}{A_s}$ may be practically horizontal, and in this case the regulation will be worst on a non-inductive instrument, and better the greater is either the lag or lead of the secondary current. In estimating the value of Mr. Campbell's experiments it must not be forgotten that the two Kelvin balances employed by him must have been of very different impedance and consequently the core inductions would have been decidedly different. This of itself would probably have been sufficient to produce the effect found. Experimental evidence will be found below on this point.

Effect of variation of frequency.—Increase of frequency increases the impedance of the instrument if inductive, but at the same time increases the E.M.F. for the same core induction. With a fully inductive instrument, therefore, and if eddy currents are absent, variations in frequency should have no effect upon the ratio. This agrees with Mr. Campbell's theoretical and experimental conclusions.

Phase Difference.—Reverting to equations (11) and (12), we have

$$\tan \phi_p = \frac{A_s \sin \phi_s + A_m}{A_s \cos \phi_s + A_c},$$

Hence

$$\tan \beta = \tan (\phi_p - \phi_s) = \frac{\frac{A_m}{A_s} \cos \phi_s - \frac{A_c}{A_s} \sin \phi_s}{1 + \frac{A_c}{A_s} \cos \phi_s + \frac{A_m}{A_s} \sin \phi_s} \quad . \quad (17)$$

which is zero for A_m and A_c both zero or for $\tan \phi_s = \frac{A_m}{A_c}$ as is obvious.

Hence for zero phase-displacement the instrument should be so inductive that the ratio of its reactance to resistance equals the ratio of the magnetizing to the core-loss currents of the transformer.

As the relation between $\frac{A}{A_s}$ and $\frac{A_c}{A_s}$ is not in general linear it follows that the phase-displacement cannot be constant, but

it is fairly evident that the best constancy would be approximately obtained when the actual displacement is smallest, *i. e.* with an inductive instrument.

With variation of frequency it is again obvious that with a fully inductive instrument, without eddy currents in the transformer, the phase-displacement should be unaffected by frequency. The less inductive the instrument the more rapidly should the phase-displacement decrease with frequency, owing to the drop in the induction density.

P.D. Relations.—Writing the secondary current in the form $A_s(\cos\phi_s + j\sin\phi_s)$ and the primary

$$A_s\cos\phi_s + A_c + j(A_s\sin\phi_s + A_m),$$

let r_p and r_s be the primary and secondary resistances and x_p and x_s the primary and secondary reactances for coils of a single turn each.

Then

$$V_p = E + \{A_s\cos\phi_s + A_c + j(A_s\sin\phi_s + A_m)\}(r_p - jx_p)$$
$$-V_s = E - A_s(\cos\phi_s + j\sin\phi_s)(r_s - jx_s),$$

from which

$$V_p = E + A_s\left\{r_p\left(\cos\phi_s + \frac{A_c}{A_s}\right) + x_p\left(\sin\phi_s + \frac{A_m}{A_s}\right)\right.$$
$$\left. -j\left[x_p\left(\cos\phi_s + \frac{A_c}{A_s}\right) - r_p\left(\sin\phi_s + \frac{A_m}{A_s}\right)\right]\right\}. \quad (18)$$

and

$$-V_s = E - A_s\{r_s\cos\phi_s + x_s\sin\phi_s - j(x_s\cos\phi_s - r_s\sin\phi_s)\} \quad (19)$$

Voltage Ratio.—Taking the ratio of $\frac{V_s}{V_p}$, simplifying, and neglecting squares of small quantities, we have

$$R_c = R_t\left\{1 - \frac{A_s}{E}[(r_p + r_s)\cos\phi_s + (x_p + x_s)\sin\phi_s] - \frac{A_c}{E}r_p - \frac{A_m}{E}x_p\right\},$$

which will be seen to be correct from the vector diagram.

Denoting $r_p + r_s$ by r the equivalent resistance,

$x_p + x_s$ by x „ reactance,

and $\sqrt{r^2 + x^2}$ by I „ impedance,

$$R_c = R_t\left\{1 - \frac{A_s}{E}(r\cos\phi_s + x\sin\phi_s) - \frac{A_c}{E}r_p - \frac{A_m}{E}x_p\right\}. \quad (20)$$

For an electrostatic or high-resistance voltmeter, $A_s=0$ and we have

$$R_v=R_t\left\{1-\frac{A_c r_p+A_m x_p}{E}\right\}. \quad . \quad . \quad . \quad (21)$$

For a hot-wire instrument $\phi=0$ and

$$R_v=R_t\left\{1-\frac{A_s r+A_c r_p+A_m x_p}{E}\right\}. \quad . \quad . \quad . \quad (22)$$

Returning to formula (20) we have

$$\frac{\Delta R_v}{R_v}=r_p\Delta\left(\frac{A_c}{E}\right)+x_p\Delta\left(\frac{A_m}{E}\right)$$

if the instrument and internal impedances are constant. This is independent of the phase of the secondary current and can only be zero if A_c and A_m are proportional to E, which is only the case for an air-core transformer. For an iron-core transformer the best constancy of ratio is thus obtained by making A_c and A_m and also r_p and x_p as low as possible, the core being of the best quality iron, and the coils subdivided and intercalated.

P.D. Phase Relations.—From equations (18) and (19) we obtain

$$\tan\psi_p=-\frac{x_p\left(\cos\phi_s+\frac{A_c}{A_s}\right)-r_p\left(\sin\phi_s+\frac{A_m}{A_s}\right)}{E+A_s\left\{r_p\left(\cos\phi_s+\frac{A_c}{A_s}\right)+x_p\left(\sin\phi_s+\frac{A_m}{A_s}\right)\right\}}A_s$$

$$\tan\psi_s=\frac{x_s\cos\phi_s-r_s\sin\phi_s}{E-A_s\{r_s\cos\phi_s+x_s\sin\phi_s\}}A_s.$$

If, as should be the case with voltage transformers, the resistance and inductive drop in the transformer are small compared with E, we have

$$\psi_p-\psi_s=\beta=-\frac{A_s(x\cos\phi_s-r\sin\phi_s)+A_c x_p-A_m r_p}{E} \quad . \quad (23)$$

as is geometrically obvious.

For the phase-difference to be zero it follows that either the resistance and inductance must be zero, or that

$$\tan\phi_s=\frac{A_s^2 rx\pm(A_c x_p-A_m r_p)\sqrt{A_s^2(x^2+r^2)-(A_c r_p-A_m x_p)^2}}{A_s r^2-(A_c x_p-A_m r_p)^2}. \quad (24)$$

In this expression if A_s is large compared with A_c or A_m $\tan \phi_s = \frac{x}{r}$, or the ratio of the reactance to the resistance of the instrument should be the same as for the transformer, and the instrument should therefore be fairly inductive.

With a non-inductive instrument

$$\beta = \frac{A_s x + A_c x_p - A_m r_p}{E}, \quad . \quad . \quad . \quad . \quad (25)$$

and if the instrument takes no appreciable current compared with A_c or A_m,

$$\beta = \frac{A_c x_p - A_m r_p}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

To find the variation of phase-displacement with voltage we have evidently from (23)

$$\Delta\beta = x_p \Delta\left(\frac{A_c}{E}\right) - r_p \Delta\left(\frac{A_m}{E}\right),$$

and this is evidently zero for an air-core transformer.

Relation between the Secondary Current and the Magnetizing and Core-Loss Currents.—If l is the length of the magnetic path, a its area of cross section, and I_s the total secondary impedance (reduced to 1 turn), we have

$$\text{Effective core E.M.F. } \bar{e} = \frac{a\bar{B}p}{10^8} = \frac{a\hat{B}p}{\sqrt{2} \times 10^8} \text{ volts per turn;}$$

$$\text{Secondary ampere turns } A_s = \frac{\bar{e}}{I_s} = \frac{a\hat{B}p}{\sqrt{2} \times 10^8 I_s},$$

from which

$$\hat{B} = \frac{\sqrt{2} \times 10^8 I_s A_s}{ap} \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Assuming the core loss to be $\eta \hat{B}^c$ ergs per c.c. per cycle:—

Total core loss

$$w_m = \frac{\eta \hat{B}^c V_n}{10^7} \text{ watts,}$$

and the core-loss ampere turns

$$A_c = \frac{w_m}{e} = \frac{5\sqrt{2}}{\pi} \eta l \left\{ \frac{\sqrt{2} \times 10^8 I_s A_s}{ap} \right\}^{(c-1)} \quad . \quad . \quad (28)$$

For the magnetizing current

$$A_m = \frac{10}{4\pi} \frac{\overline{B}l}{\mu}$$

or

$$A_m = \frac{10^9}{4\pi} \frac{l}{\mu a p} I_s A_s. \quad . \quad . \quad . \quad . \quad . \quad (29)$$

These relations are sufficient to enable the magnetizing and core-loss currents to be calculated when the total secondary impedance is known. The resistances are of course readily calculable, while the equivalent inductance of the whole transformer for a single primary and secondary winding may be calculated from the following formula, deduced from that given by Prof. Kapp :

$$L = C \frac{\sqrt{2}}{10^{10}} \frac{P}{LK^2} (3KS + T) \quad . \quad . \quad . \quad . \quad (30)$$

P being the mean perimeter of the coils in inches,
L the length of gap from iron to iron, parallel to insulation between coils,
S the thickness of the insulation in inches,
K the number of sections per coil,
T the length of the winding space,
and C a constant given as 44 for core and 55 for shell type transformers.

From personal experiment, however, the author has found that the value 44 is more nearly correct for shell type transformers.

Experimental.—As some of the conclusions arrived at are of importance, and are somewhat at variance with existing ideas, some experimental evidence will be of value. A brief reference only will be necessary here as some of these tests have already been published *, together with a description of the methods by which the results were obtained.

The principal transformer tested was made for the tests, and was of the shell type ; the core having a magnetic path of 35·3 cm. and a sectional area of 71 sq. cm. Four independent coils, each of 150 turns of No. 18 D.C.C. copper wire, were wound side by side, the resistance of each coil being 1·2 ohms. Fig. 3 shows the core, and fig. 4 the

* "Phase Displacements in Resistances and Transformers," Electrician, No. lviii. p. 160.

Fig. 3.—Isometric view of Transformer Core.

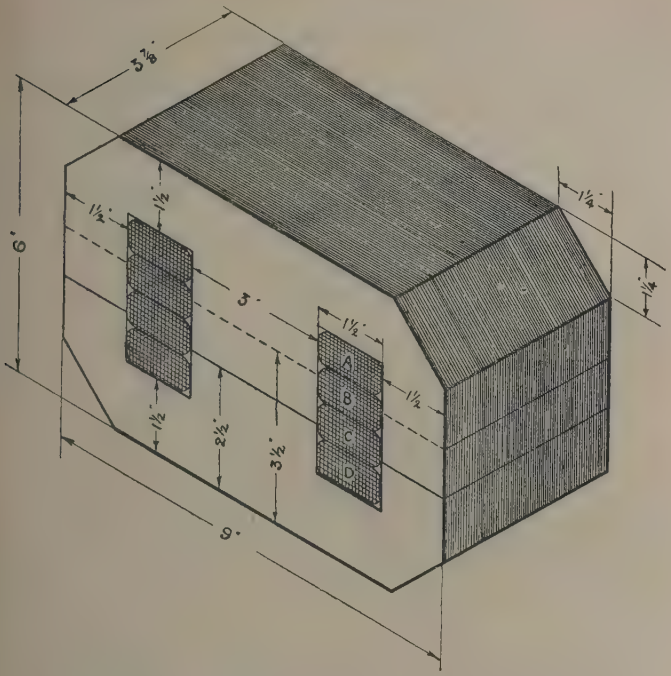
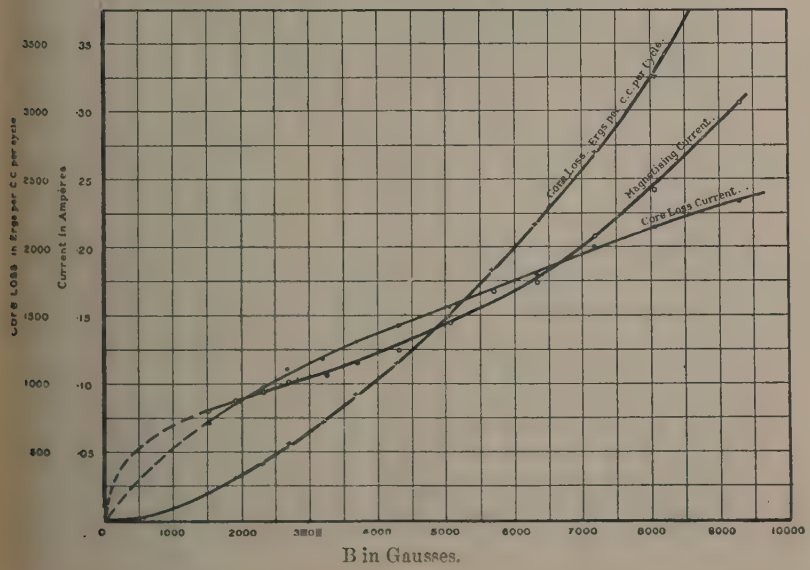


Fig. 4.—Relation of Magnetizing and Core-loss Currents to Induction.



relation of the magnetizing and core-loss currents to the core induction, determined experimentally.

Figs. 5 and 6 show the connexions for determining the current and P.D. ratios and phase displacements. The current ratio is determined by an ordinary wattmeter instead

Fig. 5.—Connexions for Current Test.

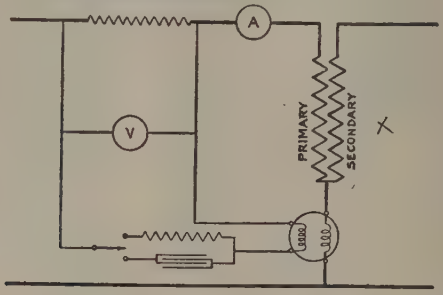
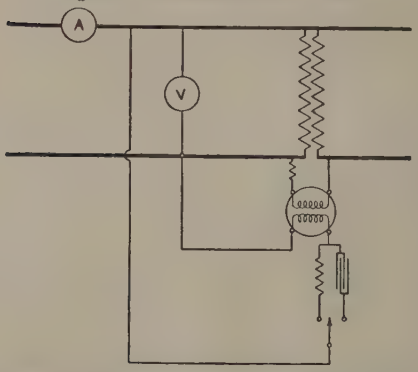


Fig. 6.—Connexions for P.D. Test.



of the differential wattmeter previously employed. Figs. 7 and 8 (Pl. IX.) show the current ratio and phase-displacements for this transformer, while figs. 9 and 10 give the P.D. relations which have been determined over a greater range of core induction than before.

As a test of formulæ 14 and 17 for the ratio and phase-displacement when used as a current transformer, we will take case VII., where the coils were separated and a secondary load of 5·9 ohms non-inductive was employed. The core induction was then found from the voltages in the unused

coils and was taken at 1565 for a secondary load of 5 amps. On measuring the equivalent reactance of the transformer it came out at 5·12 ω or 2·56 ω for the secondary leakage reactance. From this we get a total secondary resistance of 7·1 ω, reactance of 2·56 ω, and impedance of 7·5 ω approx. $\cos\phi_s = \cdot 94$ and $\sin\phi_s = \cdot 34$.

Formula 14 gives us as the loss of ratio per cent.

$$100\frac{R_t - R_c}{R_t} = 100\left\{\frac{A_c}{A_s}\cos\phi_s + \frac{A_m}{A_s}\sin\phi_s\right\},$$

which works out at 1·92 per cent., while the experimental value is 1·8 per cent.

The phase-displacement in the same case may be taken as

$$\frac{A_m}{A_s}\cos\phi_s - \frac{A_c}{A_s}\sin\phi_s,$$

which works out to ·58°, the experimental value being ·75°.

In the case of a lagging load as in curve VI. but with the coils close together, the calculated loss of ratio is 1·87 per cent. as against 2·7 per cent. by experiment, and the phase-displacement works out at ·57° * which agrees with the experimental value. The agreement between calculation and experiment is not always therefore very close, but this is probably due to the difficulty of determining the core induction. In a current transformer the magnetic leakage may be quite comparable with the main working flux, owing to the small value of the latter, and hence the distribution of flux in the core is very irregular. The mean value of the induction, however, as calculated by formula 27, agrees very fairly with the experimental value, being 1575 as against 1565 in case VII., and 1345 as against 1385 in case VI. Calculations have not been made in other cases, as the induction densities are too low.

The statement on p. 243 that a non-inductive secondary circuit should give a better ratio than an inductive one of the same impedance, is amply borne out by comparing curves V. and VI. The advantage of a low induction density is also manifest.

Calculations of P.D. ratio and phase-displacement show an

* The curves for current ratio and phase-displacement appeared in the 'Electrician' (*loc. cit.*), but in case VI. the phase-displacement was unfortunately given with reversed sign.

agreement within ·1 per cent. in the ratio and ·01° in the displacement, which is sufficient for most purposes. The theory is in each case confirmed.

Tests made with instrument transformers of various makers have in no case shown anything like such good results, and the writer is of opinion that these results represent nearly the best that can be obtained without using some special alloy of low core-loss, and also of high permeability. In view of the importance which magnetic leakage assumes in current transformers, the secondary current nearly always lags 20° to 30°, and the magnetizing current is therefore nearly as important as the core loss-current.

Fig. 11 (Pl. IX.) gives the relation between A_c and A_m and is consequently the locus of the end of the no load vector A_0 for various inductions. This vector is therefore over a considerable part of the range, at about 45°, and consequently the best ratio should be obtained with a secondary circuit leading by about 45°. In fig. 12 a curve is given for the relation of $\frac{A_c}{B}$ to $\frac{A_m}{B}$ which is of use in working out the P.D. ratio and phase-displacement.

Figs. 13 and 14 show curves of ratio and phase-displacement for three current transformers made by Messrs. Everett & Edgcumbe and Messrs. Nalder Bros. & Thompson. In the former, the primary and secondary windings were close together, giving good ratio but large phase displacement; and it is noticeable that throughout lower phase-displacement is got by greater loss of ratio, as indicated in the tests on the experimental transformer. In the Nalder transformer, the primary and secondary coils are wound on thick porcelain bobbins side by side. The leakage is therefore somewhat large, producing a variation of ratio of about 8 per cent. between 2 and 18 amperes; but this is accompanied by a phase-displacement which is less than ·2° over the range from 6 to 18 amperes or the upper two-thirds of the range. Such a transformer therefore, if used with a wattmeter, *and calibrated with it*, should do fairly well for commercial testing.

In conclusion it should be noted that the tests on all four transformers give results which are not very widely different and the results may therefore be taken as fairly representative of the behaviour of instrument transformers.

The writer wishes to express his thanks to his senior Demonstrator, Mr. A. C. Jolley, for great assistance in the experimental work, and to Mr. A. F. Burgess, B.Sc., for checking the calculations.

Discussion.

Mr A. Campbell expressed his interest in the thorough manner in which Dr Drysdale had gone into the question of the use of transformers with measuring instruments. With regard to the Author's criticism of his (Mr Campbell's) work on the subject (which was carried out twelve years ago) he should like to mention several points. The main object of the first quoted paper was to show that with air-core transformers the transformation ratio becomes more and more constant (for various frequencies) the higher we make the time-constant of the secondary circuit; thus high inductance and low resistance are wanted. Mr Campbell stated that in his paper he also gave several experiments to show that iron-ring transformers "may in many cases be used in a similar way," care being taken to have the resistance of the secondary circuit small enough. In a later paper he stated that to make the ratio sufficiently constant and independent of frequency we require relatively low resistance and high inductance in the secondary circuit. Dr Drysdale showed that for the special case of constant frequency, relatively high inductance does not give the most constant ratio. His (Mr Campbell's) experiments were not complete enough to settle this point and he was careful not to dogmatize on the matter. He was glad that Dr Drysdale had elucidated it.

Dr A. Russell expressed his interest in the paper. He suggested that the Author should take the mutual inductance between the shunt and the instrument into account. If L_1, R_1 be the constants of the instrument and L_2, R_2, those of the shunt, and if M be the mutual inductance between them, the multiplying factor for the reading will be the same whatever the frequency provided that $\frac{L_1-M}{R_1} = \frac{L_2-M}{R_2}$.

Mr Kenelm Edgcumbe referred to the fact that the Author recommended small core-losses, and pointed out that he had seen it stated that in special cases it was possible to improve the ratio and phase errors by increasing the core-losses.

Dr Drysdale, in reply to Mr Campbell, said his praise of the air-core transformer was justified, but in practice instrument-makers were forced to use iron for commercial reasons. Similarly, the ring form of transformer was generally impossible as in practice instrument transformers were used to insulate the observing instrument from high pressure mains, and considerable insulation between the windings was necessary. The reason why an iron-cored transformer behaved differently to one with an air core chiefly resided in the core-loss current which was not proportional to the magnetizing current. The method of testing the transformers would be shown in operation in the laboratories.

XV. *Experiments on Artificial Fulgurites.*

By Dorothy Deane Butcher*.

[Plates X.–XIII.]

Introduction.

Fulgurites are tubes formed by lightning striking sand or rock and fusing it. The rock fulgurites somewhat resemble worm-borings lined with a glassy substance. It is with the sand fulgurites that we are concerned.

These latter have been found in many different parts of the world—Drigg in Cumberland, Senner in Lippe, Dibla near Lake Chad, the Libyan Desert, South Carolina, Florida, and many other places. They have been found in the sand-dunes at Bondi, near Sydney, New South Wales, by Mr. J. W. Grimshaw, who has kindly given us the opportunity of examining his collection.

The fact that the tubes found are of two sorts—strong, rocky, thick-walled tubes, and smooth, cylindrical tubes with glassy walls—was pointed out by Dr. Fiedler in 1817 (1), and he suggested the possibility that one sort was formed by positive and the other by negative electricity. This idea seemed to be supported by the fact that the cross section of a thick tube shows a radial structure bearing a marked resemblance to the Lichtenberg figures made by a spark from a positively charged conductor.

It was with the view of testing this hypothesis that attempts were made to form fulgurites artificially.

Natural Fulgurites.

The thin-walled tubes are found much more frequently than the others. The former are very often squeezed together and flattened as if the tubes had been subjected to pressure when in a molten condition. This is probably due to the pressure exerted on the thin walls by the surrounding sand and the simultaneous decrease of pressure within the tube as the hot gas cools (Pl. X. fig. 5).

Of the reports of fulgurites examined only 8 localities

* Read April 10, 1908. Communicated by Sidney Skinner, M.A.

produced thick tubes, whereas there were 18 localities where thin tubes were found, and in five cases the thin tubes had retained their cylindrical form (figs. 1 2, 3). Amongst the Bondi fulgurites both thin and thick walled specimens occur.

Some of the tubes are branched whilst others appear to be unbranched, but as very few are found unbroken it is difficult to make sure of this. The tubes appear to be more or less tapering as they go down, but very few have been excavated to their ends. The fulgurite found near Dresden by Dr. Fiedler and now in the British Museum (2) is 17 feet long, and consists of 532 pieces; that found at Drigg in Cumberland was traced for over 30 feet without any sign of the end being found (3).

Amongst the tubes found near Macclesfield in a bed of sand 100 feet thick, one was removed complete, and it was found to taper almost to a point. These tubes started 12 feet below the surface and were 9 or 10 feet long, with an internal diameter 0·7 to 0·1 inch (4).

As a rule the tubes are vertical, but they are sometimes inclined at an angle of 45° to 60° either for the whole or for part of their course, and when there are branches these are occasionally horizontal or even inclined upwards. Good tubes are found only in sand. It appears, therefore, that the chemical nature of the soil determines the completeness of the tube.

In one case at Zankendorf near Vienna (5), where there was a bed of moist clay under the sand, the tube terminated in a small hollow lump with several openings in it. The course of the lightning could be clearly seen under these as long red threads penetrating the clay for about 8 inches, and radiating in all directions like fine roots. There was no fusion after the lightning left the sand, but the fact that these threads were caused by the lightning was shown experimentally by the clay turning red when heated artificially.

Microscopical and chemical analysis show that the tubes are composed of silica, being often purer in composition than the sand in which they are formed (6). The melting-point of silica is very high, but it is possible that the salts present in

the soil act as a flux and render a somewhat lower temperature sufficient (7), but even then the temperature developed by the electricity must be very high.

Artificial Fulgurites.

Since the means at our disposal make it useless to attempt to produce a large quantity of heat by a spark, some material with a lower melting-point than silica had to be used. The substance must be capable of being reduced to a fine powder, must fuse easily, and should be a poor conductor of electricity. Several substances were tried such as salt, sugar, borax, resin, and sealing-wax, but the best results were obtained with powdered glass.

This material mixed with common salt had already been used in attempts to manufacture artificial fulgurites in Paris in 1828 (8). The "Batterie du cabinet de Charles," the largest Leyden battery in Paris at that date, was used and tubes were made 25 mm. long and $1\frac{1}{2}$ to 3 mm. across. Unfortunately no drawings of these tubes are available, and the only description given is that they are "just like the natural ones, but more delicate than those found by M. Fiedler."

In our experiments we used a battery of 12 Leyden jars made of thin glass beakers 8 in. high covered with tinfoil to about $2\frac{1}{2}$ in. from the top. These were charged by a small Wimshurst machine worked by a motor.

The powdered glass or resin was placed in a hole in a block of paraffin wax, and the spark passed from a point above to a metal plate below the hole.

When the spark was passed through a mass of finely powdered resin about 8 mm. thick, a circular hole with slightly melted walls was formed, but the walls fell in when the powder was shaken. A similar result was obtained with powdered borax crystals.

The chief difficulty lay in overcoming the explosive effect of the oscillatory spark. Unless the powder was tightly packed into the wax and covered securely, it was blown about the room. With the idea of obviating this, the oscillations of the spark were damped by introducing resistance in the circuit in the form of a piece of wet string. The

result was most successful, the tubes obtained being larger and sufficiently thick to bear handling.

The resistance used was about 80 cm. of wet half-inch tape. With too long a string there was no sharp discharge but a gradual oozing of the electricity, and the precise length required seemed to vary on different days—probably because there was more evaporation from the string on dry days.

Some of the tubes were branched, but there was no definite difference between the two ends which could be ascribed to difference of polarity. The branches were always towards the plate end, whichever way the current passed (Pl. XIII. fig. 16).

The early attempts with powdered glass were unsuccessful, but this was found to be due to the powder being so coarse that the spark passed through the air between the particles without melting the powder together. With powder which had been sifted by repeated washing and allowing the larger particles to settle, fine smooth tubes were obtained—the finer the powder the smoother were the walls of the tube.

The largest tube thus obtained was 9·5 mm. long and ·5 mm. in cross section. The length could not be increased as the jars would not give a spark of greater length.

Experiments with the powder under pressure greater than the atmospheric up to 30 in. excess gave very similar results.

When the wet string was removed from the circuit no tubes could be made in the powdered glass under the ordinary conditions.

To explain this action of the wet string, experiments were made with a form of Kinnersley's electric thermometer. This apparatus consisted of a T-piece of narrow glass tubing having the tail of the T turned up again and ending at a somewhat higher level than the crosspiece. Platinum wires were sealed into the two arms of the crosspiece, and the junction of this with the vertical tube was blown out into a small bulb. The vertical tube was half filled with water and acted as a manometer tube, being open to the air at the free end.

When a spark passed between the two platinum wires, the air in the bulb was heated and expanded driving the water out of the U. The level of the water served to measure the

expansion of the air, and thus, indirectly, the heat developed by the spark. It was found that the oscillatory spark threw the water violently up in the tube, but the water immediately returned to a point which served to measure, by its distance from the original level, the heat developed. As was expected, the heat developed without the string was some three times greater than with the string. It was evident, therefore, that the repulsions between the elements of the oscillatory current prevented the formation of a tube with that form of discharge.

Once or twice when using the undamped spark, when the powder was very tightly packed into the wax cylinder and the two terminals sealed on the ends by a thick coating of wax to prevent any possibility of the powder escaping, very fine lacework tubes were obtained of much greater size than those got with the damped spark. They were about 1 cm. long and 2 to 4 mm. in diameter, but fell to pieces on being touched, the walls being a mere network of fine threads and looking more as if the particles had been stuck together, than like the result of a fusion of the particles themselves These tubes were only obtained once or twice, most attempts only resulting in blowing away the powder.

The small size of the tubes formed was apparently merely due to the want of power of the electrical apparatus; the length of the tubes which it was possible to make being approximately equal to the length of the spark which could be obtained from the jars.

We might therefore hope to get much larger tubes if we could enlarge the potential and capacity of the Leyden jars used, but those obtained are quite of sufficient size to prove the question which was under consideration.

The results of these experiments may be summarized as follows:—

1. The tubes are formed by fusion of the powder which surrounds the column of air in which the spark passes. The length and thickness of the tube depend on the energy of the spark, and also on the character of the spark, *i. e.* whether it is unidirectional or oscillatory. This latter is shown by the fact that the same quantity of energy stored in the Leyden jars does not form a tube when the discharge is abrupt and

oscillatory, but forms one if the discharge is rendered less oscillatory by the intervention of a wet string resistance, although a large portion of the energy in the latter arrangement is absorbed in overcoming the resistance of the string.

2. There is no appreciable difference in the two ends of a tube provided that the two electrodes are alike. When one electrode is a point and the other a flat plate, any branching that may occur will be towards the plate whichever electrode is made positive.

In nature, the flat plate would be represented by the moist lower strata of the soil. Therefore we cannot say from the character of the tube whether the lightning discharge was from a positive or negative cloud.

3. The difference between thick and thin tubes is due probably to a difference in the sharpness of the flash and the resulting explosive effect. When the explosive effect is great and the quantity of material melted is small, the result will be a large-bored thin-walled tube. Whether this remains circular or becomes pressed together and distorted, depends merely on whether the fused matter has time to cool before the outward pressure of the blowing has been overcome by the inward pressure on the surrounding sand or not. In Nature, the damp sand or soil probably acts as the damp string in these experiments, and consequently causes many lightning discharges to be unidirectional. In the experimental tubes the outward pressure was so great and the quantity of fused material so small, that the walls were broken through and left as a mere network.

Such network tubes much larger and stronger have also been found naturally (9), although it is more frequent to find only one or two isolated holes in the walls (figs. 7 & 9).

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Discussion.

Dr Alexander Russell congratulated the Authoress on her interesting paper. Although he had never found a fulgurite he had noticed several phenomena which seemed connected with their formation. Several years ago, during a thunder-shower on the sea, he noticed several flashes striking its surface. Where they struck puffs of steam apparently shot upwards. The appearance was not unlike the splash made by a gannet when diving. It was probably caused by the evaporation of the water in the path of the discharge. Now, a striking peculiarity about fulgurites is that they have a hollow core. The material originally filling it has probably been vaporised. It is therefore highly probable that when a lightning flash strikes sand, a puff of smoke will be shot out from the end of the fulgurite. When testing insulating materials with high voltages, a puff of smoke from the perforation is often the first indication of the breakdown. Electricians frequently notice that when a fuse consisting of a thin wire embedded in sand "blows," a tube of siliceous material is formed. When there is considerable power involved, the tube is raised to a very high temperature and, like the glower of a Nernst lamp, it allows quite a considerable current to pass through it. It would be of interest if the Authoress would investigate the conducting power of fulgurites when heated. The difference between thick-walled and thin-walled fulgurites appears to be only one of degree. If we assume that the energy to be got rid of is the same in the two cases, then, when a lightning discharge has a long path to traverse before reaching a good conducting stratum, the fulgurite would in general be thin-walled, but when the path is short it would be thick-walled. It is obvious that a very appreciable amount of energy must have been expended in making some of the fulgurites, parts of which were exhibited.

Dr C. H. Lees asked if fulgurites, when found, were hollow or filled with sand &c.

The Chairman asked if there were any changes in the magnetic properties of a material when it was formed into a fulgurite.

Mr Skinner expressed his interest in the remarks of Dr Russell on the electrical conductivity of fulgurites, and pointed out that if the sand contained salt the fulgurites would probably be conducting. The suggestion that there might be changes in the magnetic properties was interesting and might be tested by experiment.

XVI. *An Experimental Investigation of the Nature of the γ Rays.* *By* W. H. Bragg, *M.A.*, *F.R.S.*, *Elder Professor of Mathematics and Physics in the University of Adelaide, and* J. P. V. Madsen, *D.Sc.*, *Lecturer on Electrical Engineering* *.

In papers recently published in the Proceedings of the Royal Society of South Australia (May and June 1907) and in the Philosophical Magazine (October 1907) an attempt was made to show that the æther-pulse theory of γ and X rays might prove to be incorrect after all, and that most of the known properties of these rays could be explained more simply and directly on the supposition that they were material and consisted of neutral pairs. The arguments were based on a comparison of known phenomena with deductions from each of the two opposing hypotheses. At that time there did not seem to be any opportunity of appeal to a decisive experiment.

The object of this paper is to give a preliminary account of an investigation which appears to us to give the final answer as regards the γ rays, and to show that they are material in nature.

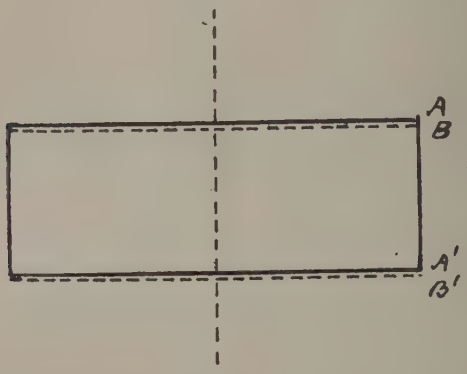
The argument is as follows:—

Secondary radiation which is excited in an atom by a passing wave or pulse must be distributed symmetrically with regard to a plane passing through the atom perpendicular to the direction of motion of the pulse. If we speak of the primary pulse as going forwards, the secondary radiation is just as likely to go backwards as forwards. This is a well-recognized principle. For example, J. J. Thomson divides the secondary radiation due to γ rays into two equal parts which he supposes to move away symmetrically in opposite directions, and, for convenience of calculation, parallel to the direction of the primary rays ('Conduction of Electricity through Gases,' p. 406). Suppose, therefore, a pencil of γ rays to pass normally through a plate so thin that its absorption may be neglected, the secondary radiation

* Read April 10, 1908.

should be exactly the same on the two sides of the plate in amount, in quality, and in distribution; and it ought not to be possible to discover, by any comparison of the secondary radiations on the two sides, which is the face of entry and which of emergence.

Fig. 1.



Consider now the ionization-chamber represented in fig. 1. The two ends are closed by plates, of which A and A′ are alike; so also are B and B′. The material of A and A′ is different to that of B and B′. The nature of the side walls is of no consequence. A pencil of γ rays passes along the axis of the chamber, which is represented by a dotted line. The ionization current within the chamber is measured as usual by inserting a high-potential electrode connected to an electroscope.

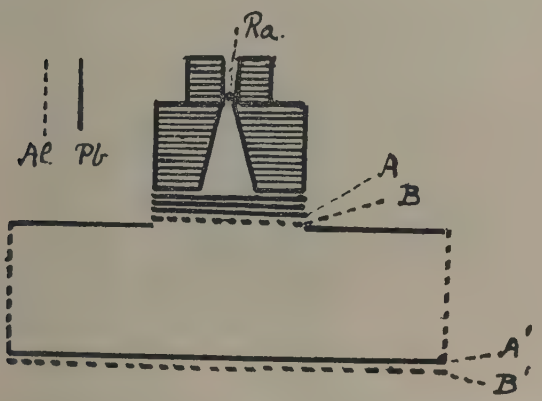
When the plates A and B are inverted there is a change in the amount of the current: so also when A′ and B′ are inverted. By an extension of the principle already stated it ought not to be possible, on the æther-pulse theory, to discover which way the rays are going (up or down in the figure) by comparing the consequence of inverting A and B with that of inverting A′ and B′.

As a matter of fact the direction can be discovered with ease; the more easily the greater the difference between the atomic weights of A and B.

For example, in one experiment of ours the chamber was

of cylindrical form, 7·5 cms. high and 25 cms. diameter. The plates used were aluminium and lead ; the thickness of each plate was a little less than 2 mm. Inversion of the top plates A and B made a difference in favour of Al of less

Fig. 2.



than 1 per cent. ; *i. e.*, the current was slightly larger when the Al was next the chamber. On the other hand, inversion of the bottom plates made a difference of 44 per cent. in favour of Pb ; *i. e.*, the current was 44 per cent. larger when the Pb was on top. The details are shown in the figure. Allowance was made for all radiation other than that which proceeded down the conical opening in the lead block.

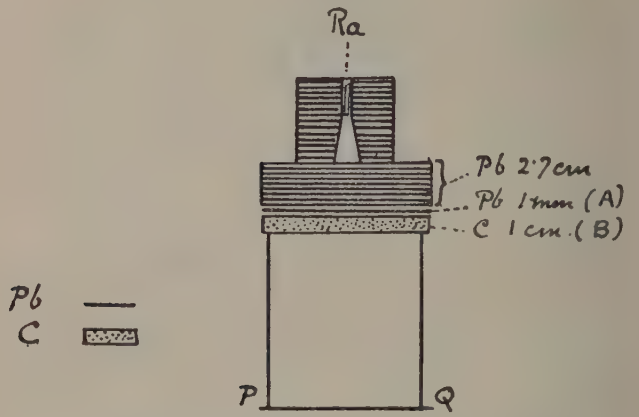
It may be well to point out that this effect cannot be ascribed to any complication due to secondary or tertiary rays. No doubt the radiation in the chamber is very complex ; but the fact is immaterial. Provided that the chamber is symmetrical in the first place, then the secondaries must be symmetrical also if the æther-pulse theory is correct, and therefore the tertiaries and so on. Nor is it necessary to consider whether the secondary radiations are β rays or scattered γ rays.

Also it must be remembered that the secondary radiations which enter the chamber have their origin almost entirely in a very few millimetres of material bordering on the chamber. Therefore the γ rays are in almost exactly the same condition,

both as to quality and as to quantity, when they excite secondary radiations from the top plate as they enter the chamber, and secondary radiations from the bottom plate as they leave.

The details of the experiment may be varied greatly; but in all the cases we have tried the want of symmetry is obvious. In fig. 3 are shown the details of one other case, in which

Fig. 3.

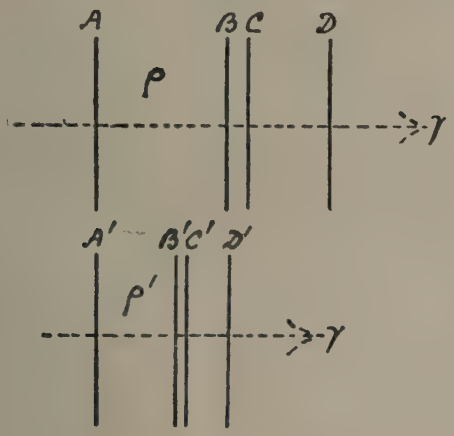


(1) Current with plates arranged as above .... 59·8
(2) „ „ A and B inverted ..... 54·4
(3) As in (1), but base PQ changed to carbon . 50·5

carbon and lead were the materials used, and the form of the chamber was different. It seems unnecessary to give more, because in the first place the experiments are easy to repeat: and in the second place, the complete quantitative analysis of the figures depends on several factors, the influence of which is imperfectly understood, such as the previous screening of the rays, the form of the chamber, and the respective parts played by the original γ rays, cathode rays, and secondary γ rays if any such exist. The experiments as they stand show how far away is that symmetry which the æther-pulse theory demands. It seems to us that there is no escape from the conclusion that the γ rays are not æther pulses.

Let us therefore proceed to consider the hypothesis that the γ rays are material. In the paper already mentioned it was argued that they might well consist of neutral pairs, liable to be broken up on encountering atoms or parts of atoms; and that the secondary cathode radiations might be the negative particles thus set free. Let us suppose, provisionally, that the particles when set free move at first in the direction of the γ stream, but are subsequently scattered in the usual manner of β rays. [It is here that the absence of symmetry arises. On the pulse theory the particles should go equally backwards and forwards; indeed, if they were ejected by atomic explosions, the result of energy accumulated by passing pulses as suggested by J. J. Thomson in the case of X rays, they would move equally in all directions.]

Fig. 4.



Wigger gives a table (*Jahrbuch der Radioaktivität*, Bd. ii. p. 431) showing that the γ rays are absorbed according to a density law pretty strictly, except for the smaller thicknesses in the case of the substances of larger atomic weight. Assume this law to hold good: and also assume for the present that the absorption of β rays follows the density law. The latter is only roughly true, of course; but we

may deal with quantities in a broad fashion first, and make the proper amendments afterwards.

We can now compare the quantities of cathode radiation which should emerge from the far sides of two plates of different densities, ρ and ρ'. Let these be represented by AD and A′D′ in fig. 4 (p. 265); and let BC and B′C′ be corresponding strata of equal weight, in fact let $AB/A'B' = BC/B'C' = CD/C'D' = \rho'/\rho$. Let the plates be crossed by equal pencils of γ rays, as shown in the figure. A certain quantity of γ radiation is absorbed in crossing BC: in the language of our present hypothesis we should say that a certain number of γ particles are stripped of their positives, and the negative remainders go on. An equal number of negatives are set free in B′C′ because the two strata are of equal weight. Of those set free in BC only a certain number emerge from the face D because of the absorption of the plate CD. Since CD and C′D′ are of equal weight a similar absorption occurs in the case of the particles set free in B′C′. Thus the same number emerges from each plate. Integrating for all effective strata, the whole cathode radiations emerging from the two plates are equal.

We thus find that if the absorptions of β and γ rays both followed the density law, the secondary cathode radiation on the far side of a plate—we may call it the "emergence" radiation—would be the same for all materials. There should be no such relation between the amount of the radiation and the atomic weight of the plate as various observers have shown to be true for the secondary cathode radiation of "incidence," a relation which is closely parallel to that found in the case of β rays.

Experiment is in agreement with this theory, for it shows that no such relation exists in respect to the emergence radiations: in marked contrast to what happens in the case of the radiation from the front sides of the plates of various materials—the incidence radiations.

It is true that the emergence radiations are not all equal, but this is to be expected, because (1) the amount of secondary cathode radiation depends, as Kleeman has shown, on the previous screening of the γ rays, (2) the β rays are not absorbed strictly according to a density law, (3) the γ rays

also depart from this law. We have made no serious attempt as yet to disentangle the effects of these various disturbing factors. In fact the task promises to be long and intricate, for it will be necessary to find out how much of the ionization in the chamber is due to each class of rays: to discover the law of distribution of the radiations in space so that the form of the chamber may be allowed for, if necessary: to find out the nature of the departures from the density law of those β and γ rays which are in question, and so on. Nevertheless the results are satisfactory, so far as we have gone. The amount of emergence radiation is found to depend on the previous screening of the rays. In one case the inversion of a C, Pb pair of plates from $\longrightarrow$ C, Pb to $\longrightarrow$ Pb, C altered the current in the ratio 1 : 1·11 when the rays had been previously screened by Pb; but in the ratio 1 : ·96 when the screen was changed to C. Again, when the rays had previously passed through an iron screen, the inversion $\longrightarrow$ Pb Fe to $\longrightarrow$ Fe Pb changed the current in the proportion 1 : 1·12, but when a lead screen was substituted for an iron one the change was 1 : 1·04. In illustration of the effect of the second disturbing factor mentioned above, we have found that, other things being equal, the substances of small atomic weight give the most secondary radiation, in a general way; and it may be no coincidence that in some cases we have found Sn and Fe to give surprisingly small amounts. This is in agreement with what is to be expected; for it is clear, on consideration of the argument already given, that the greater the β-ray absorption of a substance in proportion to its density, the less "emergence" radiation should issue from it. Some observers have found Sn and Fe to possess exceptional absorbing powers. We do not wish, however, to lay any stress upon these last observations, some of which we may not have interpreted correctly; but we mention them in order to show that the inequalities that are found to exist between the emergence radiation of various substances promise to be reducible to order as soon as the difficulties of interpretation have been surmounted.

Let us now consider the cathode radiations on the front sides of the plates. Of the cathode particles set free in BC

and moving at first in the direction of the γ rays, a certain proportion, say p, is returned by what is beyond. These move towards the face A, and a certain number of them succeed in reaching it and emerging therefrom. In the case of the other plate the proportion returned is p'; the absorption in B′A′ is the same as in BA because the weights are the same. Comparing the two plates stratum by stratum, we find that the "incidence" radiation of one plate is to the incidence radiation of the other plate as p to p'. Now p and p' are the well-known constants of the β rays.

When a stream of γ rays is allowed to fall upon a plate, the cathode radiation which issues from the place of incidence must be divisible into two parts. One consists of scattered β particles derived from the stream of such particles which was travelling with the γ rays before incidence, and which was formed during the previous transit of the screens employed—solid, liquid, or gaseous. This part is scattered to an extent which depends on the atomic weight of the plate, according to the usual (McClelland's) law of β particles. The other part is originated in the plate itself in the manner just described, and the amount of it is also regulated according to the β-ray law. When, therefore, observers have measured the secondary radiation due to γ rays, and have found a law corresponding to that for β rays, the reason of the correspondence has been that they really were measuring the secondary radiation due to β rays. Properly speaking, the secondary radiation produced by γ rays, or rather *from* γ rays, is proportional to the density of the substance traversed (*cf.* Wigger's table); and this is only another form of the law of absorption of γ rays.

The relative importance of the two parts of the incidence radiation just mentioned must depend on the circumstances of the experiment*. The researches of Kleeman (Phil. Mag. Nov. 1907) show very well how the second part, which is influenced by previous screening, modifies the effect of the first part, which is not so influenced, but which follows the law of β rays strictly.

* In a recent letter addressed by one of us to 'Nature,' too much stress was laid on the part played by the first part under all circumstances.

It is easy to show, by comparing corresponding strata at the front and back of one plate, that the incidence radiation should be somewhat less than p times the transmitted radiation,—somewhat less, because the cathode radiation which is turned back is scattered and softened in the process.

To sum up :—

On the æther-pulse theory we ought to find perfect symmetry in the secondary radiations from the two sides of a plate; but experiment shows nothing of the kind.

On the material, or neutral pair theory, the "incidence" radiations should follow the β-ray law. This is known to be the case. The "emergence" radiations should not follow the β-ray law; and experiment shows that they do not. If the density law held for both β and γ rays, and if the γ rays were homogeneous, the emergence radiations should all be equal. As already explained, experiment shows that the observed inequalities give promise of ready explanation on the ground that no one of these suppositions is quite true.

It is perhaps better not to extend the preliminary account of these experiments by any lengthy discussion of the issues arising from them. Many points that invite consideration have been discussed already in the papers first referred to. Moreover, our own further experiments are incomplete ; and their full interpretation is not yet certain. We will therefore confine ourselves to one or two questions which seem to be of special interest.

The X rays resemble the γ rays so closely that it is practically inconceivable that the two radiations should be essentially different. The secondary cathode radiations which are set free when X rays impinge on any material must therefore have been part of the X-ray stream, and must start their independent existence by moving on in the line of the X-ray motion. Their velocity is much smaller than that of the secondary cathode rays due to γ rays, and they are much more readily scattered. It may still remain an open question whether or no the X-ray stream contains æther pulses. Perhaps their existence must be supposed in order to explain

the velocity experiment of Marx, and the diffraction experiment of Haga and Windt. Possibly they are also required in order to explain Barkla's polarization experiments; but we do not think that the experiment described by Barkla in 'Nature' (Oct. 31, 1907) is in any way decisive.

It seems proper to consider a possibility that the negative particle, when it moves on in the original line of motion of the pair from which it came, retains also its original velocity. It is a striking fact that the cathode particle due to the γ rays has the same speed, very nearly, as the β particle issuing from the original radioactive material. And it looks quite unlike a coincidence that similar comparisons can be made in the case of the X rays. The secondary cathode radiations due to these rays have velocities which, at the least, are of the same order as the velocities of the cathode particles in the X-ray bulb. If we examine the table given by Innes (Proc. Roy. Soc. Aug. 2, 1907, p. 461), and if we may be allowed to adopt an interpretation differing somewhat from the author's, but more natural, it seems to us, in view of the conclusions of this paper, we find that the velocities of the electrons emitted by all the metals are practically the same, zinc being an exception because it is unable to break up the hardest rays. We find that the velocities range from about 6×10^9 to $7{\cdot}5 \times 10^9$ for soft rays, and 6×10^9 to 8×10^9 for hard rays. Remembering that bundles of X rays are very heterogeneous, the natural conclusion seems to be that the softest rays give the slowest speeds, and that the velocity of the secondary rays increases with the hardness of the X rays from which they are derived. Now the hardness of the rays grows with the speed of the cathode particles in the bulb. Is it then possible that the cathode particle is first set in motion by the electromotive force in the bulb, strikes the anticathode and picks up a positive there, becomes neutral and is now called an X ray, is subsequently stripped of the positive and becomes a secondary cathode particle, the identity of the negative remaining the same throughout and its speed invariable or nearly so? The difficulty comes in when we try to consider the part played by the mass of the positive.

And, again, may not the β and γ forms be interchangeable at times? A γ particle which had been stripped of its

positive and become a secondary cathode or β ray would be lost to measurement as a γ ray ; and we should thus have an explanation of how the γ rays are "absorbed," and why the absorption follows an exponential law. And in the same way, if a β particle picked up a positive it would disappear from view as a β particle ; it would be "absorbed."

Although we have made a few experiments with magnetic fields, we have not yet come to any conclusion as to whether or no there are γ pairs which have become loosened in the attachment of positive to negative, forming a softer and more ionizing radiation. Their existence might be suspected, since there is an analogous effect in the case of X rays ; and probably they would be found more at the back of the penetrated plate than in front of it.

A few further experimental illustrations are shown diagrammatically in figs. 5 and 6 with the explanations attached.

Fig. 5.—The upper figure shows the general arrangement. The lower figures are diagrammatic, and show the currents for different arrangements of the Pb and Al at the bottom of the chamber, and at the top with the exception of the plate through which the γ rays enter. Inverting the top plates makes little difference when the upper of the two plates at the bottom is Al ; but an appreciable difference when it is Pb, because in the latter case a good deal of secondary radiation is thrown up by the Pb, and there is a tertiary from the top plate.

The same, when the conical opening is completely filled by a Pb stopper :—

12·6 12·6 17·7 18·3

The differences show the effects of those rays only which are stopped by the Pb stopper :—

27·7 28·7 39·9 44·7

These show the effect of inverting that portion only of the

Fig. 5.

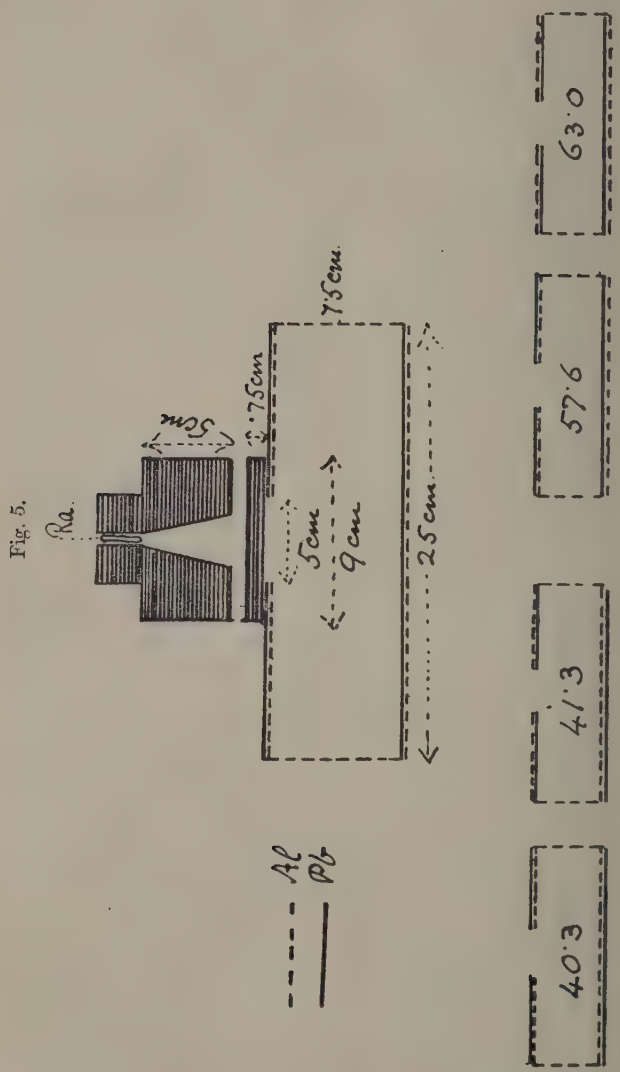


Fig. 6.

top plate where the γ rays enter. Three Pb plates = ·55 cm.; Al plate = ·16 cm.

The same, with Pb stopper inserted :—

14·1 14·7.

Fig. 6.—The upper figure shows the general arrangement. The wall of the cylindrical vessel was of brass : a Pb or an Al lining could be inserted as shown. The lower figures are diagrammatic, and show the currents for different arrangements of Pb and Al at top, bottom, and sides. Inversion of the plates *through* which the γ rays pass into the chamber makes little difference ; but there is a great change if the material is changed on which the γ rays fall, or the " emergence " radiations from the top plate. The base is of less importance than in fig. 5 ; but the sides of more importance. This should clearly be so, for geometrical reasons. When the conical opening was filled by a Pb stopper the currents were all reduced considerably, but retained the same proportions pretty nearly.

When a small pencil of β rays was admitted through a hole in the centre of the top plate a change of the material of the bottom became more effective, and of the sides less effective than before ; but this difference became smaller when thin Al sheets were so placed as to scatter the β rays on their entry into the chamber.

In conclusion we should like to add that Wigger was the first, so far as we know, to show clearly that the secondary radiation of Al, on the far side of the plate, was greater than that of Pb. A comparison of the emergence radiations of different metals was made by Dawes (Phys. Rev. xx. p. 182), who showed that they did not follow the law of the incidence radiations. The same effect was indicated in the experiments of Eve (Phil. Mag. Dec. 1904). We have little doubt that the interesting experiments of Mackenzie (Phil. Mag. July 1907) are to be explained on the lines indicated in this paper. In fact it is clear that this is the case in a broad sense ; but it is difficult to give a complete explanation until the laws are so completely worked out that they can be

applied to the interpretation of experiments which are really very complicated, although at first sight they may seem to be simple.

Discussion.

Dr R. S. Willows criticised the paper and expressed the opinion that the experiments described could all be accounted for by an extension of the æther-pulse theory. On the other hand, he instanced phenomena which would be difficult to explain on the neutral pair theory.

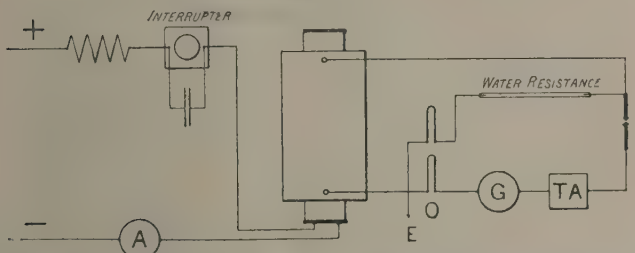
XVII. *Short Spark Phenomena.* *By* W. Duddell, *F.R.S.**

[Plates XIV.–XVI.]

In connexion with some measurements of the current in the secondary circuit of an induction-coil, I have noticed two curious effects, which are probably well known but which I do not remember having seen described anywhere.

The apparatus in use consisted of a 12-inch Newton induction-coil which was supplied from the 200 volt direct-current mains. A large resistance was placed in series with the primary of the coil to limit the current, and the current was interrupted by means of a mercury-jet interrupter; the connexions are shown in fig. 1. The secondary circuit

Fig. 1.



contained a galvanometer G to measure the mean current and thermo-ammeter T. A. to measure the root-mean-squared current.

The galvanometer was specially constructed for the purpose

* Read April 10, 1908.

so as to have a sufficiently low sensibility without using a shunt. It was of the moving-coil type and was well insulated from earth by means of porcelain insulators. The sensibility was such that 1 milliampere gave a scale-deflexion of 200 divisions (1 division equals 1/40 in.). The thermo-ammeter had a resistance of about 101·5 ohms and gave its full scale-deflexion for about 70 milliamperes R.M.S. value.

By breaking the current through the primary by means of a switch, the direction of the deflexion of the galvanometer corresponding to breaking the primary current was determined. A deflexion, in this direction I will call, in what follows, a positive deflexion, and a deflexion in the opposite direction, that is corresponding to the make of the primary current, I will call a negative deflexion.

When there was no spark-gap in the secondary circuit and the coil was in action, the mean current, as read by the galvanometer, was zero, as it should be, and the root-mean-squared current had a value of about 3·8 milliamperes.

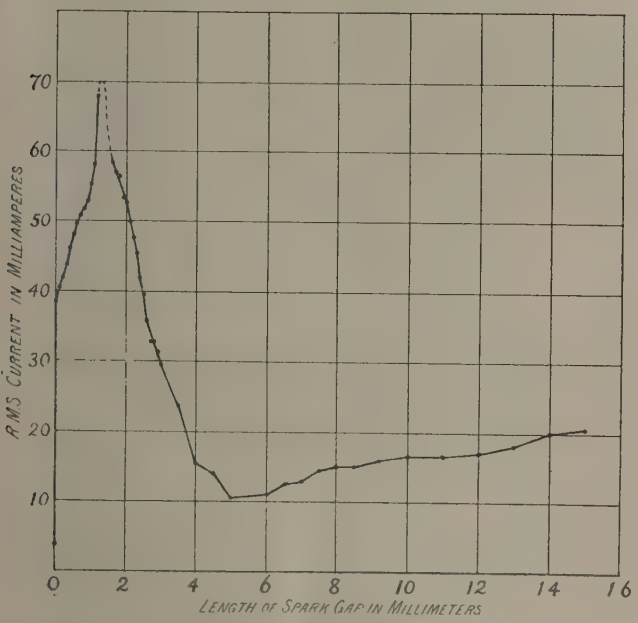
If now a microscopic spark-gap, say between two aluminium points, is introduced into the secondary circuit two curious effects take place. Firstly, the R.M.S. current enormously increases in value; and secondly, a very large deflexion is produced on the galvanometer, reading the mean current, and this deflexion is in the *negative* direction, that is to say, in the direction corresponding to making the primary circuit. I will consider these two effects separately.

In order to give an idea of the magnitude of the increase in the R.M.S. current produced by introducing a very small spark-gap into the secondary circuit of the coil, I have plotted in fig. 2 the R.M.S. current corresponding to various lengths of spark between 0 and 15 mm. In making these observations the resistance in the primary circuit of the coil and the frequency of the interrupter of the primary current were kept constant, the resistance being 137 ohms and the frequency 75 interruptions per second. The current through the primary of the coil was about 1/4 ampere.

A large number of points were taken and are plotted on the curve. They do not give very consistent results, so it is difficult to draw a smooth curve through them; but the general appearance of the phenomenon can be seen. On

introducing a spark-gap 1/10 mm. long the root-mean-squared current instantly rose from 3·8 to 38·5 milliamperes, and continued to increase with increasing spark-length until it reached a maximum at a spark-length somewhere in the neighbourhood of 1·4 mm. The exact point is slightly uncertain owing to the R.M.S. current just exceeding the range of the instrument that was in use. From 1·6 to 5 mm. spark-length the R.M.S. current gradually fell in value and

Fig. 2.



attained a minimum value of 10½ milliamperes. Further increase in spark-length produced a gradual increase in the R.M.S. current up to the maximum length of 15 mm. that was used in the experiments.

There is no doubt in my mind as to the cause of this effect. It is due to very high frequency oscillations being set up in the wires connected to the secondary circuit of the coil when a spark-gap is introduced. The magnitude of the oscillations

will depend on the voltage between the terminals of the spark-gap just before the spark passes and on the resistance that the spark-gap offers. Now the P.D. between the terminals of the spark-gap will increase with increasing length and so will the resistance, so that on increasing the spark-length we have two conflicting agencies at work, one tending to increase the magnitude of the oscillatory current, and the second tending to decrease the magnitude. I think that it is due to this differential action that the curve is such a curious shape.

The presence of the oscillations in the secondary circuit can easily be made evident by taking a well insulated metal plate and touching various points in the secondary circuit with it. The effect of this plate will be to largely increase or decrease, generally increase, the value of the R.M.S. current. The practical aspect of this question from my point of view was that, owing to the unexpectedly large value of the R.M.S. current, I burnt up several thermo-ammeters before I discovered the cause of the trouble.

I have obtained the effect with brass, iron, zinc, and aluminium electrodes and it probably takes place with all other metals. I think that, so far, the best metal to show the effect has been aluminium.

The large deflexion in the negative direction observed on the galvanometer was investigated by recording the wave-forms of the P. D. and the current by means of an oscillograph. The sensibility of the oscillograph was adjusted so that 1 mm. deflexion equals 1 milliampere for the current wave-forms. To obtain the P.D. wave-forms a water resistance of about 1 megohm was placed in series with the second moving system of the oscillograph so that 1 mm. equals about 1000 volts. The speed of the plate on which the records were taken was 1500 mm. per second. For this series of tests, the frequency of interruption was 75 per second and the resistance in series with primary of the induction-coil was 37 ohms.

Records were made for a series of spark-lengths between aluminium-point electrodes. I have selected from these some typical results which are shown in Plates XIV.–XVI. figs. 3, 4, 5, 6, 7, 8.

Fig. 3 is the current wave-form when the spark-gap is short-circuited, length 0. The straight line across the centre

of the figure is the true zero line. Deflexions above this zero line represent current in the positive direction and below the zero line represent current in the negative direction. It will be noticed that the maximum current in the positive direction (14·5 milliamperes) is much less than the maximum current in the negative direction (35 milliamperes), but the length of time that the negative current lasts is much shorter than that which the positive current lasts; so that the areas of the two sides of the zero line are equal and the mean current zero.

The smallest gap that I could make in this circuit between the aluminium electrodes, at once changed the wave-form to the type shown in fig. 4, which is for spark-length 1 mm. In this figure, the straight parts of the curve along the centre of the figure are in the position of zero current. This was carefully checked by taking records with a fixed datum-line at the zero of the curve. The line was afterwards moved to the lower part of the figure in order not to hide small details of the curve near the zero. The effect of introducing the spark-gap of 1 mm. is, while leaving the maximum current on the two sides of the zero at practically the same value, to reduce the area of the curve on the positive side of the zero line nearly to zero, so that instead of the areas on the two sides being equal there is a large excess of area on the negative side causing a large mean current in the negative direction.

On increasing the length of the gap a small area on the opposite side of the zero line again begins to form, which increases with increasing length of gap, until the condition shown in fig. 6 is reached. In this figure, the spark is sometimes rectifying or stopping the current flowing round the circuit in the negative direction altogether. We have in this figure the large triangular current wave form which corresponds to the current flowing in only one direction round the circuit, and the smaller triangular current pulses accompanied by a large current in the negative direction at make.

Further lengthening the spark-gap brings it into the normal condition of long sparks; the current wave of this is shown in fig. 7. The datum-line across the centre of the plate, in this case, is true zero line.

It was necessary in order to obtain the wave-form of these

longer lengths to disconnect the circuit for recording the P.D. wave-form, as the leak which it formed, having a resistance of only 1 megohm, prevented the sparks passing across the gap.

Fig. 8 shows the P.D. wave-form with so long a spark-gap that the spark could not jump across it. This is the normal wave-form given on the secondary of the induction-coil when supplied from a high voltage direct-current circuit.

It will be noted that the voltage induced in the circuit at the break is about 23,000 volts and at make about 13,500, so that the make voltage exceeds one half of the break voltage. Also, the voltage induced at the make dies away less rapidly than that induced at the break. It may be enquired how it is possible, if the make voltage is less than the break voltage, for the current to be larger at make than at break. I think that this question must be answered by noting that during the make period the primary of the induction-coil is connected to the supply mains so that the energy may be directly transferred from the primary to the secondary circuit by magnetic induction; that is to say, as long as we maintain a steadily increasing flow of current into the primary, we can continue to take energy from the secondary. During the break period, however, things are very different. The whole of the energy that we can get out of the secondary is that stored up in the magnetic field which is linked with the secondary winding. The greater part of this magnetic field will pass through the core. The magnetization of the core will depend upon the resultant magnetizing ampere-turns which is equal to the primary ampere-turns less the secondary ampere-turns. At the moment of break the current in the secondary is at a value of say 35 milliamperes, and the current in the primary cannot have exceeded $\frac{200 \text{ v.}}{37 \text{ ohms}}$ or 5·4 amperes.

I do not know the exact ratio between the numbers of turns on the primary and on the secondary of this induction-coil, but it is probably of the order of 100, so that the 35 milliamperes flowing in the negative direction in the secondary would correspond to a demagnetizing current of about $3\frac{1}{2}$ amperes in the primary, which would leave a comparatively small margin of resultant magnetizing ampere-turns.

If this is the case, the energy that can be got out of the secondary on break is very limited, which would account for the rapid dying away of the current to zero when even a very small spark-gap is introduced in the circuit. Directly the length of the spark-gap is sufficient to prevent the current flowing in the negative direction round the secondary circuit, the whole of the demagnetizing effect of the secondary current is done away with, and under the conditions of the experiment the magnetizing current in the primary is mainly limited by the large resistance in the circuit; hence, we get a very much larger amount of energy available directly the secondary current is prevented from flowing in the negative direction. This I think accounts for the large difference in the size of the current waves in fig. 6, and shows the great importance of preventing any current from flowing round the secondary circuit in the negative direction at make when using the induction-coil on a high voltage supply.

I made certain that the phenomenon was not due to any want of symmetry in the points of the spark-gap by reversing the electrodes of the gap and also by interchanging the connexions to it. The material of the electrodes did not seem to appreciably affect the results, but the shape of the electrodes was important in so far that the spark-length at which the galvanometer deflexion changed sign depended on the shape. Thus with two spheres the galvanometer deflexion changed sign at a shorter length than with points.

I have brought these two observations forward in the hope that in the discussion either my views as to the explanations may be confirmed or that better ones may be suggested.

Discussion.

Dr Alexander Russell thanked Mr Duddell for his valuable paper and for the exceedingly interesting and very successful demonstration of short-spark phenomena. The Author's complete analysis of the phenomena he has discovered would be a great help in elucidating the difficult theory of the action of the induction-coil and would throw light on many still unexplained effects. He believed that the change of sign of the direct current component in the secondary circuit of an induction-coil was first discovered by Mr Duddell, and the Physical Society was to be congratulated on having been the first Society to which this phenomenon was clearly described and demonstrated. He considered

that the rise in the value of the effective current when the air-gap was widened was a resonance phenomenon. The capacity *between the electrodes* was in series with the inductance of the secondary, and hence, as the Author pointed out, Kelvin oscillatory discharges were continually taking place. The shape of the curve showing the connection between the effective current and the distance between the electrodes was strong evidence in support of this view. The explanation of the direct current component when the electrodes are at considerable distances apart, may be made by remembering that the wave pulses of E.M.F. round the secondary at "make" and "break" are of very different shapes although their integral values are the same. The maximum value of the induced E.M.F. at break is much greater and the consequent rush of electricity is more "impulsive" than at make. Hence, when the electrodes are sufficiently far apart the air-gap is only broken down by the break E.M.F., and so we get a unidirectional pulsating current of triangular shape. When the electrodes are closer together we get an alternating current, the shape of the positive and negative waves being quite different owing to the differently shaped waves of E.M.F. at make and break. When they are very close together, the Author's oscillograms show that we get a pulsating unidirectional current in the direction due to the "make" E.M.F. The reason for this strange phenomenon is by no means obvious.

Mr S. Skinner expressed his interest in the paper, and referred to the experiments on the electrolysis of steam carried out by J. J. Thomson by a modification of Perrot's method. He found that with short sparks the hydrogen appeared at the positive electrode instead of at the negative as in ordinary electrolysis. Perhaps this phenomenon was due to the same causes as the effects shown by the Author.

Dr R. S. Willows asked if the maximum current obtained was connected in any way with the minimum sparking voltage.

Prof. Cassie pointed out that the minimum sparking voltage occurred at distances very much less than those used in Mr Duddell's experiments.

Mr C. C. Patterson asked if the Author had tried experiments with spark-gaps of different capacities.

Mr A. Campbell asked if it was possible that the rectifying action of the spark-gap might be similar to that of an ordinary aluminium electrolytic rectifier.

Mr Duddell said he did not think the effect was in any way connected with the minimum sparking voltage. He had tried varying the materials of the electrodes and using electrodes of various capacities and the effect was the same.

XVIII. *The Production of Small Variable Frequency Alternating Currents suitable for Telephonic and other Measurements.* *By* B. S. Cohen, *A.M.I.E.E.**

[Plates XVII.–XX.]

Introduction.—The devices described in the latter part of this paper are the outcome of experiments carried out in the Investigation Branch of the Engineering Department of the National Telephone Company, in order to obtain suitable alternating currents both of simple and complex wave form to act as substitutes for the voice in telephonic measurements. Although primarily designed for this purpose, it is considered possible that the apparatus to be described is capable of more extended use.

A short summary of the methods known to the author of obtaining small alternating currents of the kind suitable for telephonic measurements may prove of interest as an introduction.

Statement of the problem.—The ordinary telephonic current is a few milliamperes at a potential of about 2 to 10 volts and is of complex wave form. The frequency of the fundamental harmonic generally lies between 100 and 300 complete periods per second, *i. e.* it varies from 100~ to 300~, the highest harmonic having a frequency of 4000~ to 5000~, although all harmonics above 1500~ are comparatively unimportant. The average frequency of the whole wave is about 800~. From consideration of these data it follows that the ideal device for supplying such alternating current is one which will give currents of any frequency lying between 100~ to 500~, singly or in combination. The output of this apparatus should be about 1 watt, and it is necessary that it should be capable of working uniformly over a fairly considerable period of time.

Simplicity and portability are also highly desirable.

Alternators.—A sine wave alternator coupled to a motor, the combination being capable of running at a constant speed for a considerable period, is a very useful form of apparatus

* Read May 22, 1908.

for investigations in connexion with telephony. A suitable machine is, however, difficult to obtain.

The Western Electric Company build a machine with an output of about 30 watts at frequencies varying from 800~ to 1800~, and the wave form is stated to resemble a sine curve closely at all loads.

Messrs. Siemens and Halske also make a machine with an output of 3 or 4 watts at about the same frequencies. Both these machines are of the inductor type, the purity of the wave form being secured by the shape of the teeth and pole faces.

Mr. Duddell's work with high frequency alternators is well known. He has also made several medium frequency machines, one of which, with a wound rotor, gives an output of about 20 watts, but only runs up to about 500~.

In the Investigation Department of the National Telephone Company we have built a small alternator of the inductor type on the lines of the Siemens machine. This is illustrated in Pl. XVII. fig. 1.

With a 30 volt battery connected to the field windings, the output is only about ·3 watt, but the wave form is fairly good, *i. e.* it approximates to a sine wave. By decreasing the air gap, the output can be considerably increased at the expense of the purity of the wave form.

With the best air gap (about ·015 mm.) the P.D., which is independent of the frequency, is 10·5 volts.

Fig. 2 (Pl. XVII.) shows the wave form of this machine when the frequency is 800~. Measurements of the capacity of small mica condensers made with the current from this alternator on the assumption that it is a pure sine wave, are accurate to within about 3 per cent., and this forms a very good test.

For more accurate measurements this machine has been used with a wave filter, consisting of series inductances of low effective resistance and parallel capacities, similar to that described by G. A. Campbell (Phil. Mag. March 1903). The wave form when at a frequency of 800~ this filter is used is shown in fig. 3 (Pl. XVII.).

Humming Telephone.—The interaction between a receiver and a transmitter setting up an alternating current whose frequency depends on the free period of the apparatus

can be utilized. The method, however, is untrustworthy, and the output small.

The wave form is also far from sinusoidal, as might be expected when it is considered that it is influenced by the action of the polarized electromagnet in the receiver. Fig. 4 (Pl. XVII.) shows the wave form using the local battery type of telephone instrument.

Mr. A. Campbell's modification of the humming telephone consists of a steel bar with a free period of the frequency desired, and this is set in motion by a polarized electromagnet, which has attached to it a light microphone, which maintains the interaction and supplies current to the external circuit through a transformer. This apparatus is more reliable than the humming telephone. The output, however, is limited by the transmitter, and the wave form is similar to that of the ordinary humming telephone *.

Organ Pipes etc.—Organ pipes, other wind instruments, and tuning forks sounding in transmitters, were experimented on for a short time, and might under certain conditions give satisfactory results. It is difficult to avoid trouble, however, with all such arrangements, owing to irregularities introduced by the transmitters and to their limited output.

Vibrating Wire Interrupters.—The vibrating wire interrupter described by Wien, Orlich, Campbell, and others, has when suitably modified given very satisfactory results, and I propose to describe a special form of this apparatus †. It has been found very difficult to construct a vibrating wire which will run reliably at a frequency much above 300~, and I have been unable to find references to wires which could be maintained in vibration at a frequency higher than 500~. The output is also very limited, and the frequency difficult to determine and vary.

* For further particulars of these devices see Note by R. Appleyard, Elect. Rev. pp. 57 & 656, vol. xxvi. 1890; Paper on Humming Telephones, F. Gill, Journal I. E. E. vol. xxxi. 1901; A. Campbell, Proc. Royal Soc. p. 208, June 12th, 1906; J. E. Taylor, Journal I. E. E. p. 396, vol. xxxi. 1901; F. Dolezalek, *Zeitschrift für Instrumentenkunde*, p. 240, Aug. 1903.

† (1) Max Wien, Wied. *Ann.* xlii. p. 593 (1891).
do. do. xliv. p. 681 & p. 689 (1891).
(2) Orlich, *Electrotechn. Zeitsch.* vol. xxvi. (1903).

In order to overcome these drawbacks the instrument illustrated in fig. 5 (Pl. XVIII.) has been designed. The upper figure shows details of mercury cups and electromagnet, and the lower figure gives a general view of the whole instrument. The wire is steel, 1·06 mm. in diameter, and supports a soft iron armature which is maintained in vibration by an electromagnet with a laminated core of stalloy, which is an alloy of iron much used in electrical work, a mercury cup and a platinum wire contact. The wire is provided with a tension adjuster, and will run steadily at frequencies varying from 100~ to 250~.

The frequency of the alternating output is quite independent of the wire frequency, and is obtained by means of a separate circuit, mercury cup, and contact. The mercury cups are made from glass tube 4 mm. in internal diameter, and these are enclosed in brass tubes to which the mercury is connected by means of a platinum wire fused into the glass.

The cups thus made slip into brass adjustable carriers from which they can readily be removed for cleaning purposes, and the carriers admit of a fine vertical adjustment.

Experiments have been made with cups of varying diameter. Those with the smallest diameters gave the best results.

It would appear that the mercury in the small cups remains much steadier under the action of the vibrating contact than in those of larger diameter, owing possibly to the greater surface-tension effect.

Single Action Circuit.—Several forms of oscillating circuit have been used of which the two principal ones may be described as "single action" and "double action" respectively.

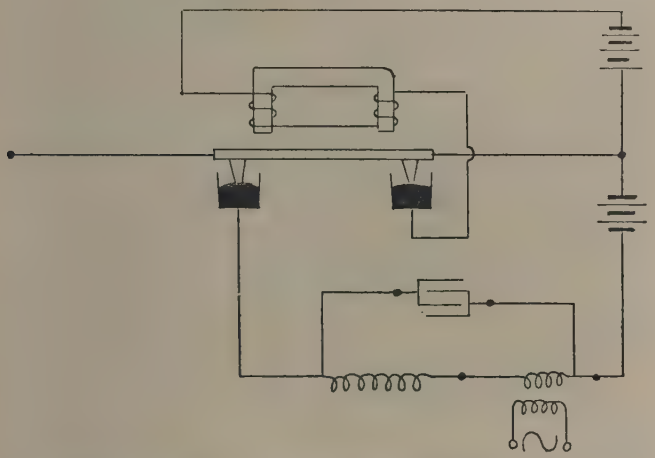
Fig. 6 shows the single action circuit and also the wire driving circuit. The latter requires no explanation. The former consists of a platinum contact and mercury cup which in the make contact position connects a battery to a capacity in parallel with an inductance and transformer primary.

On the break between the cup and contact an oscillation occurs in the circuit.

By this method a series of damped trains of oscillations of

any frequency can be produced, the trains following each other with the frequency of the wire vibrations.

Fig. 6.—Single Action Circuit.



Theory of Action.—Oscillations will occur when $\frac{R^2}{4L} < \frac{1}{K}$, their frequency being given by $\frac{1}{2\pi}\sqrt{\frac{1}{LK} - \frac{R^2}{4L^2}}$. With the values of L, K and R used, the last term can be neglected and formula becomes $\frac{1}{2\pi}\sqrt{\frac{1}{LK}}$.

The total output in watts, excluding all losses, will be the energy stored, divided by the interval of time between each succeeding train of waves.

The equation to the line joining the peaks of the damped waves is $Y = e^{-\frac{aR}{2L}}$, so that for small damping L must be large and R small, which also gives the maximum output.

In one of the circuits used the inductance is ·037 henry, and its effective resistance which is practically invariable over the range of frequency used is 10 ohms. (These figures include the resistance and inductance of the primary of transformer.)

Fig. 7 (Pl. XVIII.) shows an oscillogram of damped wave trains produced in the manner just described.

The three frequencies are 490~, 959~, and 2880~ produced by using the three capacities, 2·88 mfd, ·8 mfd, and ·1 mfd respectively.

The frequency of the vibrating wire was 90~ in all three cases. It will be observed that only part of a complete train is utilized in the lower frequency waves.

Problem of Output.—The transformer used in this circuit is a small one of the telephone type, with a few turns and a somewhat loose coupling. By reason of this, variations in the frequency and damping of the output caused by variations of the load on the secondary affecting the effective self-induction of the oscillating circuit are rendered small, but at the same time the output is reduced.

In cases where the load is constant it is possible to use a more efficient transformer and so obtain a considerable increase in the output.

It is worthy of mention that in the apparatus as constructed, the chief storage of energy occurs in the coil and not in the condenser. The single action circuit has proved of value for telephonic purposes.

Double Action Circuit.—The double action circuit previously referred to is illustrated in fig. 8.

This circuit, unlike the single action one, gives a pronounced oscillation on the make contact, as well as on the break.

On referring back to fig. 6 it will be seen that no oscillation can occur on the make contact as the battery short-circuits the oscillating circuit during that period.

By introducing resistance into the battery circuit, however, an oscillation on the make contact can be obtained in addition to the break oscillation. This oscillation is of the frequency of the main oscillating circuit.

Although the results might have been predicted theoretically, the following series of oscillograms (Pl. XIX. fig. 9) are inserted as they show in a rather interesting manner the effect of insertion of resistance.

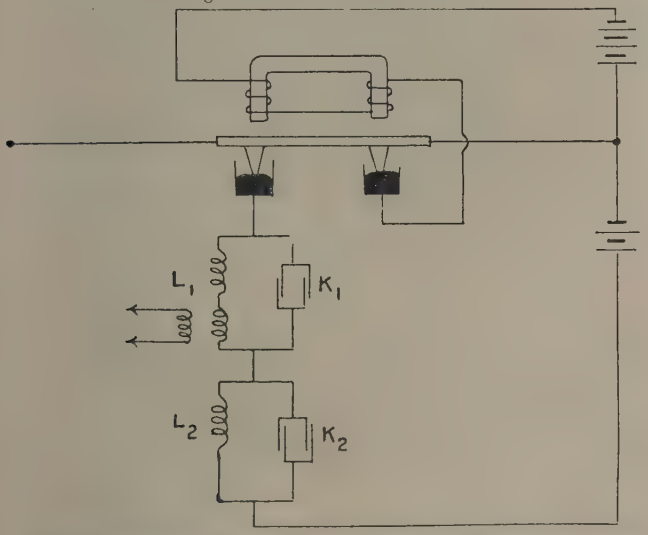
This series shows the variations obtained in the make contact oscillations in a circuit as shown in fig. 6, when the resistance in series with the battery (which is of negligible internal resistance) is varied from 4000 to 0 ohms

The make oscillation is most evident when the resistance is reduced to 1000 ohms.

At 300 ohms the oscillation begins to appear during the current rise on the make contact, whilst at 100 ohms the oscillation is entirely confined to the rise during the make contact.

The resistance is still large enough to cause the current to rise very quickly to its maximum value, thus giving a nearly square shouldered wave.

Fig. 8.—Double Action Circuit.



As the resistance is still further reduced the current takes longer to rise to its maximum value and at the same time the oscillation is reduced, until ultimately at 0 ohms the familiar current rise curve with no superimposed oscillation is produced.

In the double action circuit as illustrated (fig. 8) a capacity shunted by an inductance is introduced in place of the simple resistance. This somewhat modifies the action of the circuit. If the inductance L_2 is inserted as shown in the figure and the capacity K_2 omitted, the effect is to give

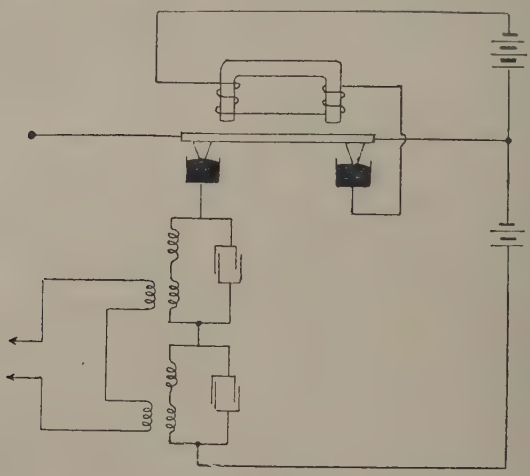
an oscillation on the make contact, the frequency of which is determined by the combined inductance of L_1 and L_2 and the capacity K_1. When the capacity K_2 is added a similar effect is produced. The resulting make frequency being due to the combined effect of both inductances and capacities. The break oscillation will be that due to L_1 and K_1 only.

Fig. 10 (Pl. XIX.) shows two waves produced by this circuit. The following small table gives the necessary data:—

Wave.	Values of				Frequency of oscillation :	
	L_1 henrys.	L_2 henrys.	K_1 mfds.	K_2 mfds.	make contact.	break contact.
a	·037	·037	1·67	·167	1020~	715~
b	·037	·037	·167	1·67	1020~	2160~

Double Action Circuit with two Transformers.—By inserting a second transformer in circuit as shown in fig. 11, two

Fig. 11.—Double Action Circuit with Two Transformers.



separate and distinct oscillations can be obtained. If the circuits are similar two separate waves will be produced the shape of which will be exactly similar.

By joining up the two secondaries so that these two waves either assist or oppose each other, some interesting results are obtained.

Fig. 12 (Pl. XIX.) shows two 900 ~ break oscillations assisting one another. The make oscillations oppose each other and are therefore wiped out, the result being to give a wave exactly similar to that obtained with the single action circuit, but with about double the amplitude.

When the secondaries are joined up so that the break oscillations oppose each other, whilst the make oscillations assist, the result, if these oscillations are equal in frequency but unequal in amplitude, is to give a continuous and more or less uniform wave.

The first wave in fig. 13 (Pl. XIX.) shows the effect of opposing two 850~ oscillations, and the second the effect with two 490~ oscillations. The continuous waves resulting are 850~ and 490~ respectively.

The regularity of the continuous wave can be best modified by slightly varying one of the inductances, which have air cores by inserting in the cores one or two strands of fine iron wire.

It is obvious since the output depends on the difference between the two circuits that it will not be large. To increase the output, different inductances for each of the two circuits can be used, and the capacities adjusted so as to get the same frequency in each circuit.

It is of course impossible to obtain a pure sine wave by this method, and the extent of the impurity has not yet been examined analytically. Some tests have, however, been made by using these waves for measurement purposes. These are given further on.

Complex Wave Form.—It is worth noticing that a number of double action circuits can be operated by means of a single mercury-cup and contact. From fig. 11, it will be seen that any number of oscillating circuits can be joined in series. The oscillations which ensue on break are quite independent, each having the frequency of its particular circuit.

The make oscillations will depend on the total capacity and inductance of all the circuit.

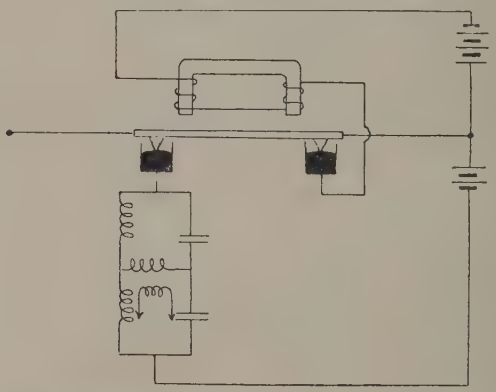
The secondaries of the transformers can be joined up in

series or in parallel to suit the external circuit conditions, and can also be joined to oppose or assist each other. By this means complex waves of definite formation can be built up.

Fig. 14 (*a*) Pl. XX. shows only the make oscillations when three oscillating circuits were used, and (*b*) gives the resulting complex wave, including both make and break oscillations. The three break frequencies are 415~, 590~, and 1442~ respectively.

Modified Double Action Circuit.—Fig. 15 shows a modified form of double action circuit which has been found to give a greater output when obtaining continuous oscillations by opposition than the circuit previously described.

Fig. 15.—Modified Double Action Circuit.



The two transformers are replaced by a single one put in such a position that both oscillations pass through it.

Fig. 16 (Pl. XX.) shows continuous waves of 1020~ and 3700~ produced by this method.

The variation in the three 1020~ waves shown are produced by the insertion of iron in the core of one of the inductances.

(*a*) No iron.
(*b*) One strand, } No. 22 S.W.G. Soft Iron Wire.
(*c*) Two strands, }

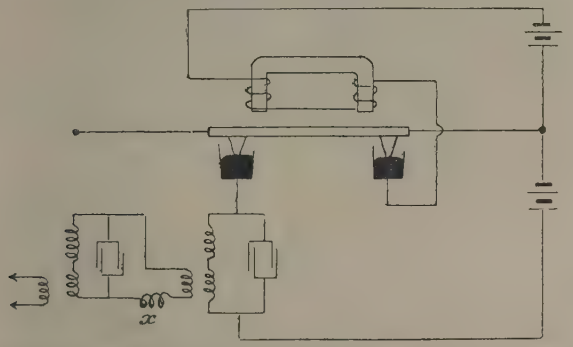
The effect of superimposing an Oscillation on an Oscillatory Circuit of similar period.—If the oscillations produced by

one circuit are superimposed on a second similar circuit, as shown in fig. 17, a continuous oscillation results.

Fig. 18 (Pl. XVII.) shows the resulting wave forms when each of the two circuits is tuned to 830~, (*a*) being the applied oscillation, and (*b*) the resultant. The *modus operandi* appears to be as follows :—

The damped oscillation applied to the second circuit causes resonance in the latter, and the oscillation set up will be in the reverse direction of the original oscillation inasmuch as it will tend to rise to a maximum whilst the latter is falling.

Fig. 17.—Continuous Oscillation Circuit.



By suitable adjustment of a series resistance, or preferably an inductance placed at *x*, therefore, it is possible to obtain in the second transformer a wave which is the resultant of two equal and opposite trains of waves. This resultant wave is practically continuous.

Mercury Contact effect.—A phenomenon, in connexion with a vibrating-point contact in a mercury cup, has been noticed to which I have not been able to find any reference. I refer to a series of sparkless points which occur at regular intervals as the distance between the contact and the mercury is varied. With the vibrator described in this paper the contact point has an amplitude of motion of about 4 mm.

If the cup is adjusted whilst the contact is vibrating until the point just makes contact on the mercury surface, a spark is observed.

If then the distance between the mercury and the contact

is slowly decreased, a series of sparkless points will be reached which are very sharply defined, as when such a point is reached, a variation either way of about ·07 mm. gives the sparking condition again.

The writer has not so far made any investigation into the reason for this action.

It is, however, most probable that this action is partly mechanical as at the sparkless points the vibrating surface of the mercury seems to be set in a rigid condition.

Use of continuous Waves for Absolute Measurements.—The wave forms illustrated in fig. 13 (Pl. XIX.), fig. 16 (Pl. XX.), and fig. 18 (Pl. XVII.) might conceivably be accurate enough for use when making absolute, as opposed to merely comparative, measurements.

The following table gives some results arrived at by measuring the current and p.d. with standard condensers subjected to these waves.

Wave form used.	Frequency.	Standard Capacity tested.	Percentage error.
Wave similar to that illustrated in fig. 13	1442	·167	−1·8
	854	·33	−1·7
	523	1·0	−1·0
Wave similar to that illustrated in fig. 16 (*a*)	833	·167	+3
	1442	·167	+4·2
Wave similar to that illustrated in fig. 18 (*b*)	833	·33	+2·4
	833	·167	+1·8
	1440	·167	+ ·5

These results were determined by inserting a non-inductive resistance of known value in series with the condenser, taking the potential-difference across both by means of a sensitive electrostatic voltmeter, and then calculating the capacity by the formula $A \cdot 10^{16}/pV$, where A and V are the effective values of the charging current and potential-difference respectively. This is very satisfactory as very little time was spent in adjusting the waves.

So far as these tests go it would appear that the amount of

the departure from the sine shape in these continuous waves is comparatively inconsiderable.

Telephonic Measurements.—The special form of vibrating wire described was primarily designed for telephonic measurements; and although the tests carried out in this direction are far too extensive to describe in any detail in this paper, a brief outline may perhaps be given.

Telephonic measurements are now mainly carried out by expressing the attenuation of speech-waves in any line circuit or apparatus in terms of the attenuation over a given length of a certain type of cable line settled on as a standard.

It has been found that the words representing the numbers 1, 2, 3, 4, 5, embody all the frequencies of telephonic importance, and these numbers are invariably used for testing purposes.

Inspection of the oscillograms of these five words as spoken by a number of persons shows that they may be approximately represented by fig. 19 (Pl. XVII.), which is given by the equation

$$y = \cdot 29 \sin pt + \cdot 32 \sin 2pt + \cdot 39 \sin 3pt + \cdot 55 \sin 4pt + 1 \cdot 06 \sin 5pt + 6 \cdot 5 \sin\left(6pt - \frac{\pi}{2}\right) - 1 \cdot 06 \sin 7pt - \cdot 56 \sin 8pt - \cdot 39 \sin 9pt - \cdot 32 \sin 10pt - \cdot 29 \sin 11pt,$$

where $p = 2\pi \times 145$.

The general resemblance of this wave to some of those produced artificially, see figs. 7 and 12, is obvious on inspection.

It has been found that by replacing the human voice and transmitter by the vibrating wire producing these waves, and by using the correct frequencies, similar results to those produced by the voice can be obtained.

Having obtained a satisfactory substitute for the human voice, it is next necessary to replace the ear and receiver by some measuring instrument.

Simple measurements of the comparative current attenuation over the standard line and the line or apparatus under test do not give accurate results, and it was suggested by my assistant Mr. A. J. Aldridge, to whom I take this opportunity

for expressing my indebtedness for considerable help given in carrying out the investigations embodied in this paper, that direct measurements of the comparative volumes of sound issuing from the telephone receiver could be made by allowing the receiver to sound into a transmitter connected up in the usual manner to a battery and induction-coil, and to measure the current in the secondary of the induction-coil.

This method has been found to solve most of the difficulties, and with the vibrating wire at the sending end of the line and at the other end of the line a receiver sounding into a transmitter, the latter being connected to a barretter used as an alternating current milliammeter, it has been found possible to carry out comparative telephonic measurements with greater accuracy than with the voice and ear, owing to the elimination of the personal equation and the greater sensibility to volume variation of the apparatus used.

This method can be used for such diversified tests as comparison of loaded with unloaded lines, comparison of standard and non-standard apparatus, such as receivers, transmitters, induction-coils.

Conclusion.—Apart from telephonic measurements, the modified vibrating-wire interrupter may possibly have a field for both comparative and absolute electrical measurements, and especially for use with the vibration galvanometer. The ease with which the frequency can be varied together with the steadiness of the resulting wave and the simplicity of the apparatus are certainly great recommendations. The author must express his indebtedness to the National Telephone Co. for the facilities afforded to him, to Dr. A. Russell for great help in the revision of this paper, to Mr. A. Campbell for references to humming and vibrating wires, and a tribute must also be paid to the oscillograph which has been of such great assistance in these investigations.

Discussion.

Mr A. Campbell mentioned that with his vibrating-bar microphone hummer he had found it easy to obtain more than one frequency from the same bar by altering the supports to suitable nodal points and tuning the magnet circuit with a condenser. Harmonics of the fundamental

frequency are thus produced. He asked if any of the composite wave forms shown gave vowel sounds in the telephone.

Dr Russell congratulated the Author on his interesting and valuable paper, and thanked him for the instructive demonstration he had given of methods of producing high-frequency currents suitable for telephonic and other measurements. He understood that the Author's main object was not the obtaining of currents of excessively high frequency, but obtaining currents having frequencies varying between 100 and 5000 which followed the harmonic law. Some of the methods described, in particular the Author's "double action circuit" method, seemed well adapted for this end. In some of these circuits there are two free periods of vibration, and it was exceedingly interesting to see the accuracy with which the oscillograph gave the resultant of two slightly damped trains of high-frequency waves of different periods. Tuning for resonance the Author obtains a very perfect sine wave. He thought that the thanks of the Meeting were also due to the National Telephone Company for the facilities they had afforded Mr. Cohen.

Mr. Cohen expressed his interest in Mr. Campbell's statement that it was easy to get more than one frequency from his vibrating bar hummer. The currents shown, when passed through a telephone receiver, did not produce vowel sounds.

XIX. *A Vacuum-Tube Model for Demonstrating the Propagation of Alternate Currents in Cables.* *By* Dr. C. V. Drysdale *.

[Plates XXI. & XXII.]

As is well known, the propagation of electric waves along conductors can be conveniently shown on the small scale by means of a helix, fed from a suitable source of high frequency, high potential oscillations. In this case the helix is found to glow with brush discharges at the antinodes of potential, remaining dark at the nodes. Experiments of this kind have been shown by Prof. J. A. Fleming and by G. Seibt †. The glow is, however, very faint, and may be rendered much more visible by the use of a vacuum-tube, as Prof. Fleming himself suggests.

When experimenting lately in this direction, the present writer noticed that if a long vacuum-tube was laid parallel to

* Read March 27, 1908.

† See 'The Principles of Electric Wave Telegraphy,' by Dr. J. A. Fleming, pp. 251–261.

the helix, light and dark portions were observed, but that these did not coincide with the points of maximum and minimum potential respectively; and it was of course evident that such a tube is influenced not by the actual potential, but by its slope or $\frac{\partial V}{\partial x}$ along the tube. As the intrinsic equation of propagation is $\frac{\partial V}{\partial x} = IC$, it is clear that the brightness of the tube is proportional to the current in the helix, and that therefore the whole distribution of current and potential can be exhibited by vacuum-tubes fixed parallel to and perpendicular to the helix respectively. A few photographs of the effects observed were published in a recent paper *, but these were obtained by successive exposure of the same tube in different positions.

In order to show these effects with the greatest convenience the model shown in fig. 1 (Pl. XXI.) has been made. The helix is 182 cms. long and 2·65 cms. in external diameter. It is wound on a glass tube with 25·5 turns per cm. of 13 mil. D.S.C. wire; and has a resistance of ·43 ω, an inductance of $\frac{\pi^2 d^2 n^2}{1000} = 44$ microhenrys per cm. The capacity from experiments with a metal tube of similar diameter appears to be $9{\cdot}2 \times 10^{-6}$ mfd. per cm.

This helix is supported in a wooden frame, and has above it a carbon-dioxide vacuum-tube 1 inch diameter. Immediately below the helix and about 2 mm. from it are the spherical bulbs of 18 vertical carbon-dioxide tubes, which are mounted in brass sockets on a metal strip fixed to a shelf at the bottom.

The helix is excited from a 10 in.-spark induction-coil with an oscillation-circuit consisting of six leyden-jars of about ·002 microfarad each, which can be connected in series or parallel; and an adjustable helix having a maximum inductance of ·000365 henry.

In order to exhibit the effect of different receivers at the end of the line a water resistance, consisting of a tube about 5 mms. diameter and 30 cms. long, was made up, as well as

* "The Theory of Alternate Current Transmission in Cables," Electrician, vol. lx, No. 10, p. 358.

an adjustable parallel-plate condenser about ·7 sq. ft. in area, and an inductance-coil of 110 yards of No. 16 G.P. covered wire in a hank about 10 in. diameter. By means of mercury cups and paraffin blocks, various arrangements of leyden-jars and loads could be easily made.

The photographs, figs. 2 to 9 (Pl. XXII.), are of the apparatus, and of the effects observed when currents of different frequency were employed, and different terminal conditions. The sending end was in each case on the left. In figs. 3, 4, and 5 the frequencies were about 600,000, 1,140,000, and 1,800,000~ per sec. respectively, giving one, two, and three half-waves. In each case the current and potential waves are out of phase; and at the higher frequency, where resistance is of less effect, they are almost exactly in quadrature, as required by theory.

The remaining figures are all with the low frequency and should be compared with fig. 3. Fig. 6 shows the effect of earthing the receiving end through the water resistance; and that, as theory indicates, the current and P.D. become much more uniform in magnitude. In fig. 7 the receiving end is earthed, resulting in a shifting of the waves to the right, so that a second current-loop appears.

Fig. 8 illustrates the effect of increasing the capacity of the helix by introducing an earthed wire inside it, and shows that with the same frequency the wave-length is decreased, so that there are two half-waves instead of one.

Finally, fig. 9 shows the effect of earthing the receiving end through a condenser. The current-loops are shifted forward as when the receiving end was earthed, and at the same time they are much more definite. With inductance the same result is observed, but the shift is backward, as the theory indicates.

No attempt has as yet been made to check the results by theory with great accuracy, nor does it seem probable that this will be of much value, as the presence of such a large number of vacuum-tubes alters the capacity and introduces leakance into the helix. But the general effects observed are of value in illustrating the theory of wave-propagation.

The author's thanks are tendered to Mr. C. M. Dowse who has taken considerable trouble in arranging the apparatus and

experiments, and to Mr. A. C. Cossor for the construction of the vacuum-tubes, which were only obtained sufficiently uniform after a considerable amount of trouble.

Discussion.

Mr W. Duddell expressed his interest in the experiments, and asked why the points of zero potential were not more clearly marked. Was it due to absorption of energy by the tubes or to the superposition of oscillations with higher frequencies? Had the Author used a single long tube with a strip of tinfoil on the outside? He also asked what gas the Author used in the vertical tubes, and whether he had tried helium.

Dr Drysdale in reply said that the tubes were filled with CO_2. It was somewhat difficult to ensure their equality, and no attempt had therefore been made to use rare gases. Doubtless still better results could be obtained, but the CO_2 tubes were sufficiently bright for most purposes. He believed that the energy taken by so many tubes was the cause of the absence of complete darkness at the nodes, as with a single tube sharper results had been obtained. For this reason the single tube with a strip of tinfoil on the side had not been used.

XX. *An Elementary Treatment of the Motion of a Charged Particle in a Combined Electric and Magnetic Field.* *By* W. B. Morton, *M.A.*, *Professor of Natural Philosophy, Queen's College, Belfast* *.

One of the methods employed by Prof. J. J. Thomson in his fundamental work on the ratio of charge to mass of a corpuscle† depends on the theorem that a charged particle, liberated without appreciable velocity at a metal surface, and moving under the combined influence of a constant electric force, urging it at right angles to the surface, and a uniform magnetic field parallel to the surface, will describe a cycloid. The result comes at once by integration of the equations of motion of the particle, but in view of the importance of the matter it may be worth while to point out that it can be established in a geometrical way without requiring anything more advanced than a knowledge of the velocity and acceleration of a point on a

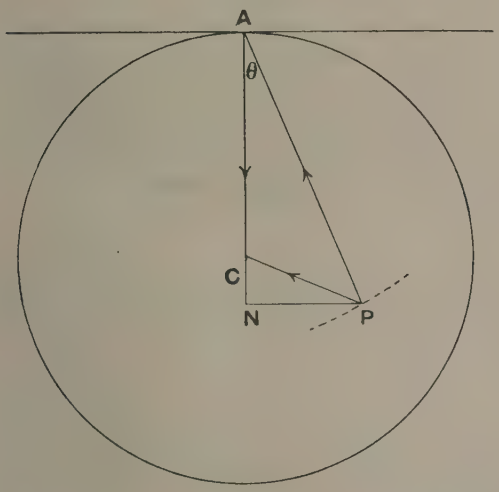
* Read June 12, 1908.

† J. J. Thomson, Phil. Mag. [5] xlviii. p. 547 (1899).

wheel which is rolling steadily along a straight line. It is only necessary to show that the acceleration of such a point can be resolved into two components, one of constant magnitude and direction, perpendicular to the line along which the circle rolls, and the other perpendicular to the instantaneous direction of motion of the point and proportional to its velocity. These correspond to the accelerations given to the corpuscle by the electric and magnetic fields respectively. If the point is on the circumference of the circle its velocity will vanish when it is the point of contact with the fixed line, so its initial circumstances coincide with those of the corpuscle liberated at the plate. Therefore their paths are identical.

By taking the tracing point anywhere in the plane of the circle, we get the more general result that a corpuscle projected in any manner in a plane perpendicular to the magnetic lines, will describe a trochoid.

Fig. 1.



Let P (fig. 1) be any point in the plane of the circle which is rolling along the horizontal line with constant angular velocity ω.

Then A is the point of zero velocity,

C the point of zero acceleration.

Therefore the velocity of P = its velocity relative to A = ω.AP, in direction perpendicular to AP, and the acceleration of P = its acceleration relative to C = ω^2.PC along PC.

Replace this by

ω^2.PA along PA and
ω^2.AC parallel to AC.

This is evidently the resolution required, for the latter component is constant in magnitude and direction, and the former is $\omega \times$ (velocity of P) and is perpendicular to this velocity.

On identifying the accelerations with the electric values we obtain at once the size of the circle and its rate of motion.

Using a for the radius and v for the velocity of P at the instant

$$\omega^2.a = \frac{Ee}{m}$$

$$\omega^2.PA = \omega v = \frac{Hev}{m},$$

where the charge e on the particle, the electric force E, and the magnetic force H are expressed in absolute electromagnetic units.

Hence ωa = velocity of rolling = E/H

and $$a = \frac{Em}{H^2 e}.$$

It is the height of the cycloid ($2a$) which is determined in Prof. Thomson's experiments as the greatest distance from the plate reached by the corpuscles.

It will be seen that we have now the following representation of the motion of a corpuscle projected from the point P with velocity v in a given direction perpendicular to the lines of H. Draw a line PA at right angles to the direction of projection (towards the side to which H exerts its force), measure off a length $v/\omega = vm/He$ to the point A. Describe a circle of radius Em/H^2e with A as its highest point, and let this circle roll, along the horizontal tangent at A, with

velocity E/H, carrying the point P along with it. Thus the size and motion of the circle are independent of the manner of projection.

In further confirmation of the agreement between the motion of the charged particle and this geometrical representation, it may be noticed that the mechanical force arising from H, being always perpendicular to the motion, does no work on the particle. Thus the kinetic energy of the latter is derived wholly from the work of the electric force, and we have

$$\tfrac{1}{2}mv^2 = \mathrm{E}ey + \text{constant};$$

where y is the vertical distance below the horizontal plane.

In the rolling circle we have

$$\begin{aligned} v^2 = \omega^2 . \mathrm{PA}^2 &= \omega^2\{\mathrm{AC}^2 + \mathrm{CP}^2 + 2\mathrm{AC}(\mathrm{AN} - \mathrm{AC})\} \\ &= 2\omega^2 ay \quad + \text{constant} \\ &= \frac{2\mathrm{E}e}{m} y \quad + \text{constant} \end{aligned}$$

in agreement with the last.

When P is on the circumference of the circle we get the cycloidal path, the corpuscle coming to rest on the fixed straight line. If P is outside the circle the trochoidal path is looped, and the corpuscle crosses the fixed line when its direction of motion is vertical. When P is inside the circle the path is a wavy one, at mean distance a below the line.

It is easy to express the condition which determines the form of path, in terms of the magnitude and direction of the velocity v. Let θ be the angle between v and the horizontal $=\mathrm{PAC}$ in figure 1.

Then we have a looped path if

$$\mathrm{PA} > 2a\cos\theta,$$

i.e.

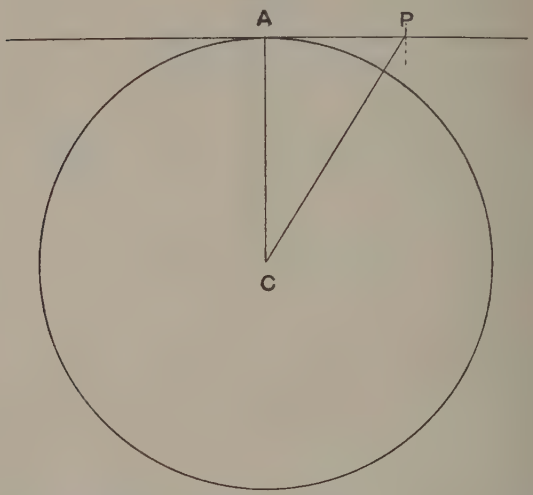
$$v > 2\,\frac{\mathrm{E}}{\mathrm{H}}\cos\theta.$$

So the path is looped or waved according as the velocity of projection is greater or less than the component of the velocity of the lowest point of the circle, resolved along the direction of projection.

It seems of some interest to find the greatest distance from

the metal plate which would be reached by a corpuscle projected normally with velocity u, say, instead of being liberated without velocity. In this case P is outside the circle, and the plate takes the place of the line along which the circle rolls.

Fig. 2.



Taking P on this line we have (fig. 2)

$$u = \omega \,.\, \mathrm{AP},$$

$$\therefore \qquad \mathrm{CP} = \sqrt{a^2 + \frac{u^2}{\omega^2}}.$$

The maximum distance reached is

$$a + \mathrm{CP} = a\left\{1 + \sqrt{1 + \frac{u^2}{a^2\omega^2}}\right\} \text{ instead of } 2a$$

$$= \frac{m}{e\mathrm{H}^2}\{\mathrm{E} + \sqrt{\mathrm{E}^2 + u^2\mathrm{H}^2}\}$$

Thus the correction is of the second order in the ratio of the velocity of expulsion to the velocity E/H. The latter velocity in the experiments described by Prof. Thomson (*loc. cit.*) was of order 10^8 cm. sec.$^{-1}$.

There is an alternative mechanical way of describing the motion in the cycloidal case. It can be shown without difficulty that it is identical with the motion of a cycloidal pendulum, or of a particle sliding on a smooth cycloidal wire, the range of swing extending from cusp to cusp. Here the acceleration of gravity takes the place of the electric force, and the normal reaction of the wire or string that of the magnetic force. But this analogy cannot be extended to the trochoidal motion; the velocity of the moving corpuscle does not alter in the same way as that of a particle sliding under gravity on a smooth trochoid.

23rd May 1908.

XXI. *A Directive System of Wireless Telegraphy.* *By* E. Bellini *and* A. Tosi*.

Part I.—Bilateral Directive Wireless Telegraphy using Closed Oscillatory Circuits: (i.) for Transmission; (ii.) for Reception; (iii.) for both Transmission and Reception.

Part II.—New Unilateral Directive Method: Simultaneous Use of Closed Oscillatory Aerial Circuits and the Open Oscillator or Vertical Antenna.

The authors, having lately obtained some interesting and very remarkable results in the course of their work upon a further development of their directive system, propose to give an account of the arrangements employed and the results attained.

Since the complete understanding of these experiments depends upon the properties shown by the closed oscillatory circuits used by the authors, it is considered desirable, in the first part of the paper, to recapitulate, as briefly as possible, the main features of the original method, thus enabling a more comprehensive view to be taken of the later arrangements, as well as supplying the necessary information to those who are not familiar with the earlier work, which has been fully dealt with elsewhere †.

* Read June 12, 1908.

† Electrical Engineering, ii. pp. 771–775 (1907), and iii. pp. 348–351 (1908). The blocks referring to this part of the paper have been kindly lent by the Kilowatt Publishing Company.

Part I.

The aerial of the bilateral system is composed of vertical closed oscillatory circuits.

The closed oscillatory circuit is, as is known, an oscillatory circuit, the metallic portion of which forms a geometrically "nearly closed" figure, the electric field of which is concentrated in a very narrow region, and the conditions of vibration of which are independent of the position of the spark.

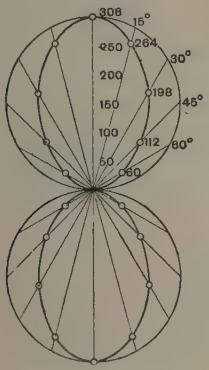
Although in such circuits the electric field is situate almost exclusively between the armatures of the condenser, none the less there always exists an electric field of dispersion the lines of force of which go from different points of the circuit to neighbouring conductors and to earth. Consequently, if one of these circuits be vertically placed in proximity to the ground, and with the condenser at the upper part, a number of lines of force will reach the earth, giving rise to an electromagnetic field which propagates itself in space. Since one of the armatures of the condenser has at the same instant an equal potential of opposite sign to that of the other, the electromagnetic field of one side will be opposed in phase relative to the field of the other side. As a result it is evident that in the direction perpendicular to the plane of the circuit there is no radiation whatever; that is to say, the said type of circuit radiates principally in its own plane.

An oscillatory circuit placed in an electromagnetic field will become the seat of an E.M.F. due to the variation of magnetic flux across the surface bounded by the circuit. This E.M.F. will vary with the cosine of the angle which the plane of the circuit makes with the direction of propagation of the field.

Using the Duddell thermogalvanometer, the authors have determined the diagrams of the energy emitted or received by such a circuit. The energy diagrams obtained were equal and composed of two equal tangent curves (fig. A) whose equation is $E = E_0 \cos^2 \alpha$, where α is the angle which the direction of propagation makes with the plane of the

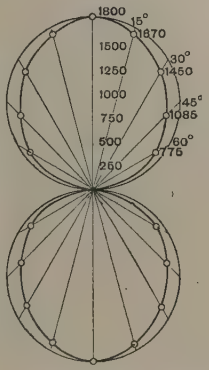
circuit. The diagram of the electromagnetic field is composed of two equal tangent circles (fig. B).

Fig. A.



Energy Transmission Diagram (Micro-watts).

Fig. B.



Transmission Diagram of Electromagnetic Field Intensity (Microamperes).

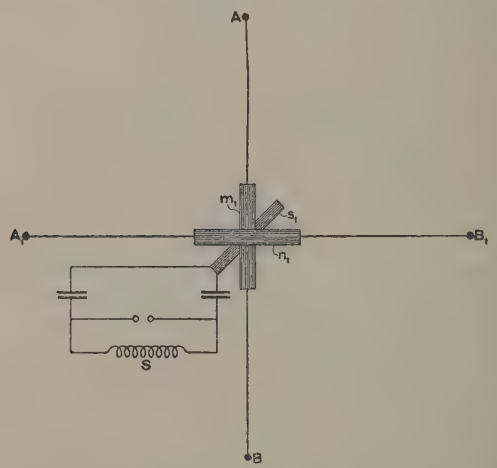
It is owing to these properties possessed by closed oscillatory circuits that such circuits were chosen by the authors for use as directive aerials; and the triangular form of circuit, open at the apex, was employed, owing to the fact that it could easily be sustained by a single mast. In order to be able to transmit and receive to or from any direction whatsoever, without having to turn the aerial, or having recourse to a large number of aerials in fixed positions, the aerials were built up of two equal closed oscillatory circuits, vertically placed and mutually perpendicular. This aerial system is connected up to special instruments for transmission and reception which the authors have called "Radiogoniometers."

The radiogoniometer for transmission comprises two fixed windings, m_1, n_1, mutually perpendicular, one of which is inserted in each of the two aerial circuits AB, A_1B_1, as shown diagrammatically in horizontal projection in fig. C. A third winding s, enclosed by the two first, is arranged so

as to be movable on its vertical axis, which axis is that of the intersection of the planes of the two fixed windings. This winding s is included in the oscillatory exciting circuit, by means of contact brushes. S is the secondary of the spark coil or transformer.

It is evident that on placing the movable winding in different positions the magnetic flux which passes through the two fixed windings will vary, as also will the oscillatory currents in the aerials. The electromagnetic field in space will be produced by the superposition of the partial fields created by the two aerial circuits. Its intensity will be

Fig. C.

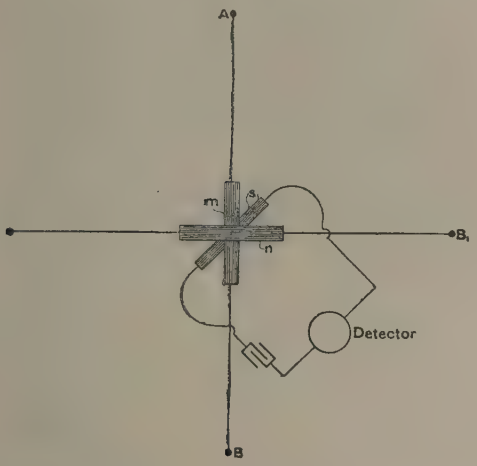


independent of its direction, and this latter will coincide with the direction of the central plane of the movable winding. The law of angular variation of the energy and of the magnetic field is the same as for a single circuit.

The apparatus is provided with a fixed dial over which moves a pointer rigidly attached to the movable winding. On setting the pointer in the direction of the station to which the signals are to be sent, the emission will be a maximum in the vertical plane which contains the transmitting and receiving stations—that is to say, both towards the receiving

station, and at 180° from this. The receiving radiogoniometer is similar in construction to that for the transmission. The fixed windings *m*, *n* (fig. D) are connected to their respective aerial circuits. The movable winding s_1 is suitably connected up to the detector, through two brushes.

Fig. D.



The oscillatory currents set up in the two aerials, when these latter are subjected to the influence of an electromagnetic field, create, in the interior of the fixed windings, a resultant magnetic field of an intensity which is independent of the direction of the transmitting station. When, therefore, the movable winding is perpendicular to this resultant field, that is, its central plane is oriented towards the transmitting station, the effect on the detector will be a maximum. In this position the reception is practically limited to the plane containing the transmitting station, and the pointer indicates on the dial the direction of this plane. One is, however, not able to determine *which* side of the receiving station the transmitting station is located.

The receiving radiogoniometer is provided with special contacts for tuning the fixed windings to the lengths of the waves to be received, which is effected by varying the number

of active turns equally and simultaneously on the two windings.

The experiments with the system have been carried out at three stations on the French side of the English Channel, at Dieppe, Havre, and Barfleur respectively. The distance Dieppe-Barfleur is 107 miles, all over sea; that between Dieppe and Havre 54 miles, entirely over land; the angle made by the three stations Havre-Dieppe-Barfleur is 23°.

Employing this bilateral system it has been possible to transmit from Dieppe to either Havre or Barfleur without the emission which was directed towards the one post troubling the other one. The energy sufficing for this purpose was about 500 watts.

By the use of the receiving radiogoniometer at the Dieppe and Havre stations the authors have been able to determine the direction of stations both known and unknown. Further, by receiving the same transmission (signals) simultaneously at both Dieppe and Havre, and determining at each station the bearing of the unknown station, it has been possible to locate, by intersection, the actual position of the station which was transmitting. The accuracy obtainable by the use of the receiving radiogoniometer is about one degree of arc.

The triangular closed oscillatory circuits comprising the aerial system were 45 metres high, with a base of 55 metres.

Part II.

The system of wireless telegraphy reviewed in the first Part allows of transmission and reception in the plane in which the station with which one is communicating is situated. That is to say, in transmitting, one is sending the signals to the corresponding station and at the same time in the opposite direction also. As regards the reception, either of two diametrically opposite transmitting stations will affect the receiver in the same way, and it will not be possible to decide as to which side of the receiving station the transmitting station is actually located.

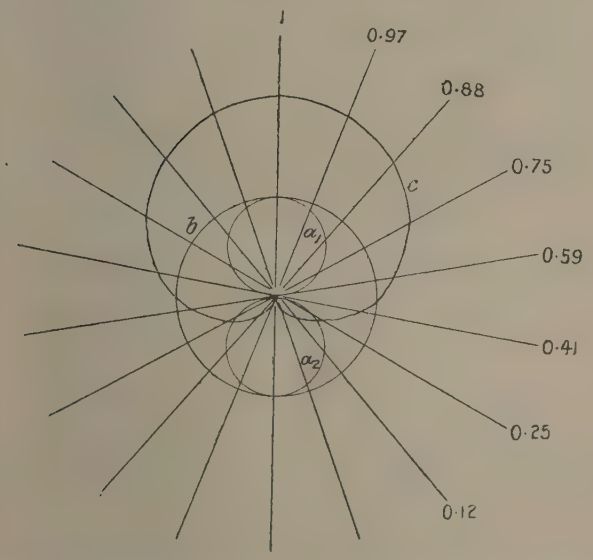
Since the sending of the waves in a single useful direction

is often of great importance, and it is desirable to be able to decide not merely the direction but the azimuth, the authors have worked out a method of achieving this object, and have solved the problem in the following manner.

Transmission.—The diagram of the electromagnetic field of the bilateral directive system already described is composed of two equal tangent circles, and the one half-field, represented by one of the circles, is opposite in phase to the other half-field.

If one operates simultaneously, at the same station, both the directive system and an ordinary system giving a circular emission, whose vertical antenna is symmetrically placed with reference to the directive aerial, the resultant electromagnetic field of the simultaneous emissions of the two systems, can, in general, be obtained by vectorially adding the partial diagrams of the two systems.

Fig. 1.



As an example, let a_1 and a_2 (fig. 1) be the two tangent

circles forming the diagram of the electromagnetic field of the directive system; the circumference b represents the diagram of the system with circular emission, of amplitude equal to the maximum amplitude of the directive system; let the phase-difference between b and a_1 be zero, and that between b and a_2 180°. The resultant diagram c is obtained in this case by adding algebraically the *radii vectores* of the curve a_1 to those of the diagram b and by subtracting the *radii vectores* of the curve b from those of curve a_2. The resultant diagram is represented by a "cardioid."

It is evident from the figure, that the resultant maximum electromagnetic field is directed to one side only; that it has a value double that of the component electromagnetic fields; and that the field on the other side is zero.

The intensity of the electromagnetic field set up by the directive system at a point whose direction makes an angle α with the maximum radiation, is expressed by $C \cos \alpha$. Let M be the intensity of the electromagnetic field produced by the circularly radiating system at the same point, and let ϕ be the phase-difference between the electromagnetic field of the circular system and that of one side of the directive system.

The intensity of the resultant electromagnetic field at the point considered will be

$$I = \sqrt{(M + C \cos \alpha \cos \phi)^2 + C^2 \cos^2 \alpha \sin^2 \phi}$$
$$= \sqrt{M^2 + C^2 \cos^2 \alpha + 2MC \cos \alpha \cos \phi}.$$

The minimum of I with reference to α is obtained when

$$\cos \alpha = -\frac{M}{C} \cos \phi.$$

This value of $\cos \alpha$ is imaginary when $M \cos \phi > C$.

In the case when $M \cos \phi \leq C$, one has $I_{min} = M \sin \phi$; when $M \cos \phi > C$ one has

$$I_{min} = \sqrt{M^2 + C^2 \cos^2 \alpha - 2MC \cos \alpha \cos \phi}.$$

In the special case when $\phi = 0$ the equation of the resultant

electromagnetic field transforms itself into

$$I = M + C \cos \alpha,$$

which is the equation of a curve that can have three different forms according to the value of the ratio M/C. The curve represented by the condition M = C is the cardioid above mentioned.

But since in wireless telegraphy the action depends chiefly upon the energy, it will be useful to consider this in preference to the intensity of the electromagnetic field.

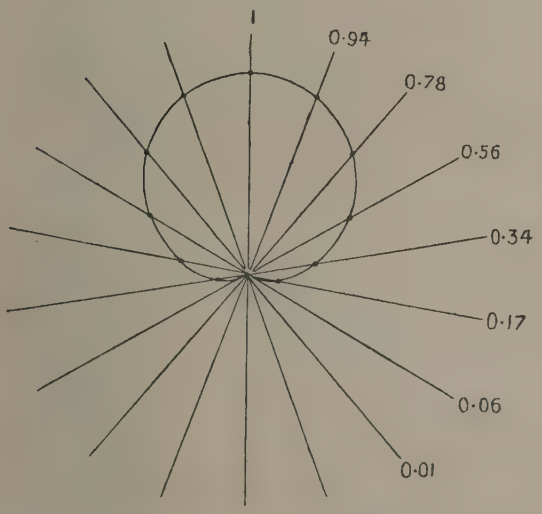
In the general case the energy radiated in the different directions is expressed by the equation—

$$W = M^2 + C^2 \cos^2 \alpha + 2MC \cos \alpha \cos \phi,$$

and in the case of the cardioid by the equation—

$$W = M^2(1 + \cos \alpha)^2,$$

Fig. 2.



The corresponding curve is given in fig. 2.

Fig. 3 represents the energy diagram in the case where $M/C=0{\cdot}72$ and $\phi=53°$.

Fig. 3.

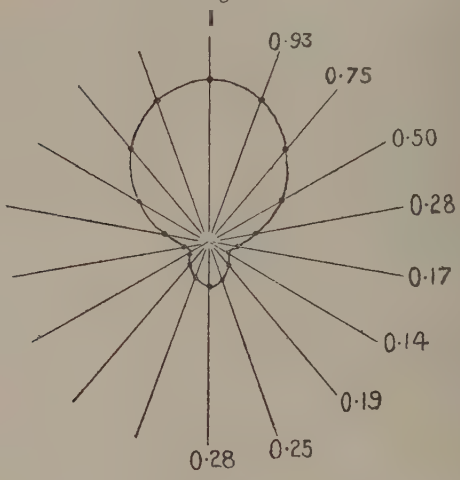


Fig. 4.

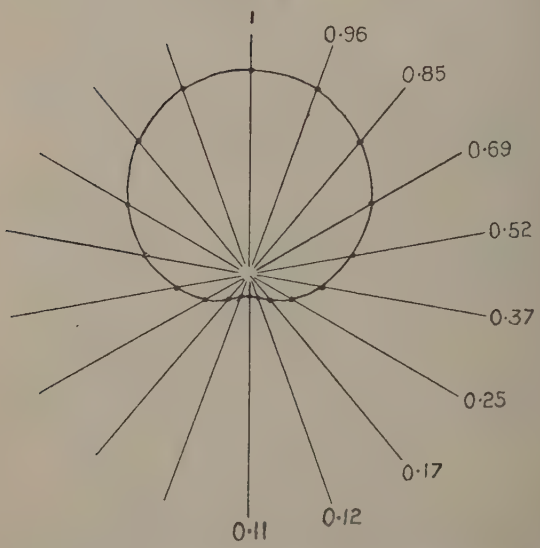
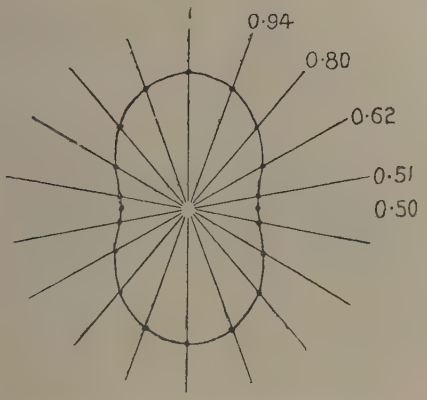


Fig. 4 shows the same diagram for the case where $M=2C$

and $\phi = 0$, and finally fig. 5 the same diagram for M = C and $\phi = 90°$.

Fig. 5.



In consequence, one can conclude that the shape of the diagram of the energy radiated in the different directions depends upon the value of the phase-difference of the component electromagnetic fields, and that the superposition of a circular system on a directive system allows of the concentration of the emitted energy in the desired direction and in the useful sense only.

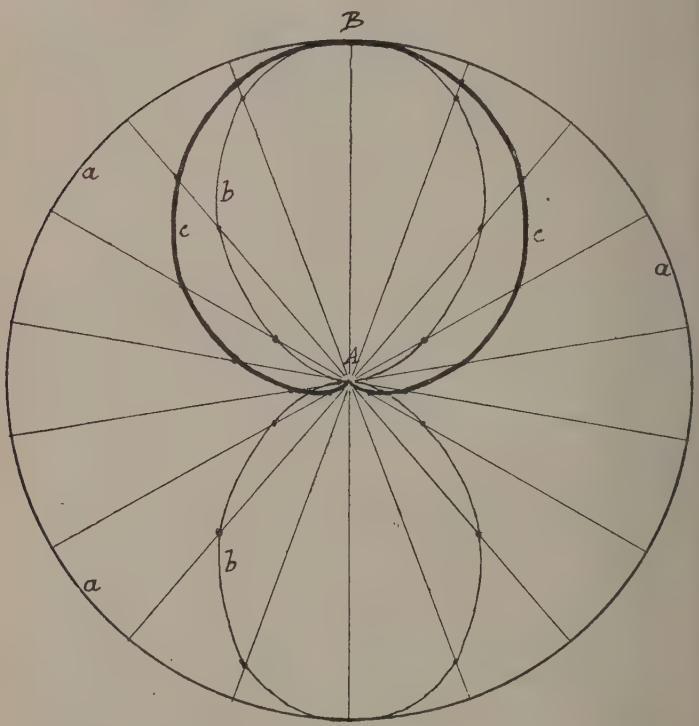
Fig. 6 (p. 316) shows diagrammatically the advantages which the employment of the mixed unilateral system has over the ordinary circular, or the simple bilateral directive method.

Let it be supposed that a station A ought to transmit to a station B. If the station A uses a circular system the energy is radiated all over the circular region a, affecting all the stations comprised within this region. If A uses the bilateral directive system the active area is the region bounded by the two tangent curves b, b. If the station uses the mixed unilateral system adjusted for M = C and $\phi = 0$, the active area is considerably smaller than that with the other methods.

The practical realization of the superposition of the two systems has been effected by employing as the directive

system the bilateral system already dealt with in Part I. It was evident *a priori* that, owing to the different conditions under which the radiation from the directive circuit takes place with reference to the radiation from the vertical antenna, a phase-difference between the emissions of the two systems should exist, for equality of phase in the excitations.

Fig. 6.



As will be seen further on, the experiments have shown that this phase-difference is 90° or near to that value. The aerial of the unilateral system is formed by the aerial of the directive circuit to which has been added a vertical antenna in a position symmetrical with reference to the first. To secure the simultaneous excitation of the closed oscillatory circuits and of the

vertical antenna, various arrangements have been employed which are diagrammatically depicted in figs. 7, 8, 9, 10.

Fig. 7.

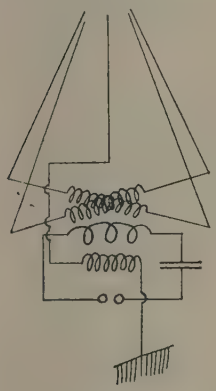


Fig. 8.

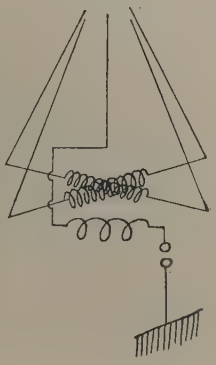


Fig. 9.

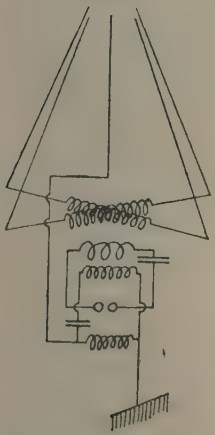
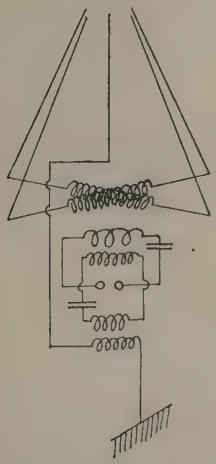


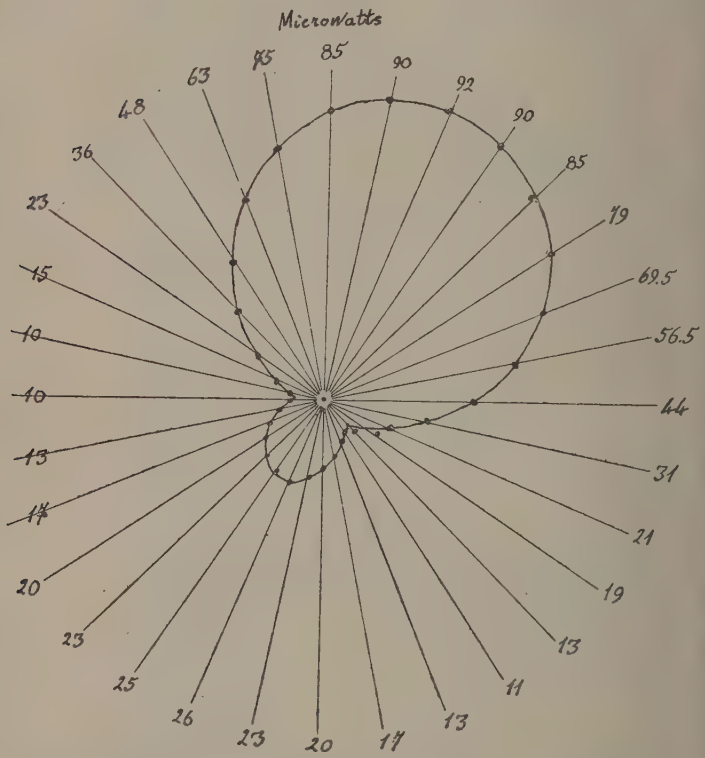
Fig. 10.



In the first of these arrangements (fig. 7) the excitations of the two systems are in phase; in the other three cases they are in quadrature. The diagrams of the energy radiated in the different directions (figs. 11 and 12) are selected from

among the large number which have been determined. They were obtained with the thermo-galvanometer in a similar manner to that employed in the case of the bilateral directive system. In particular the diagram of fig. 11 was obtained

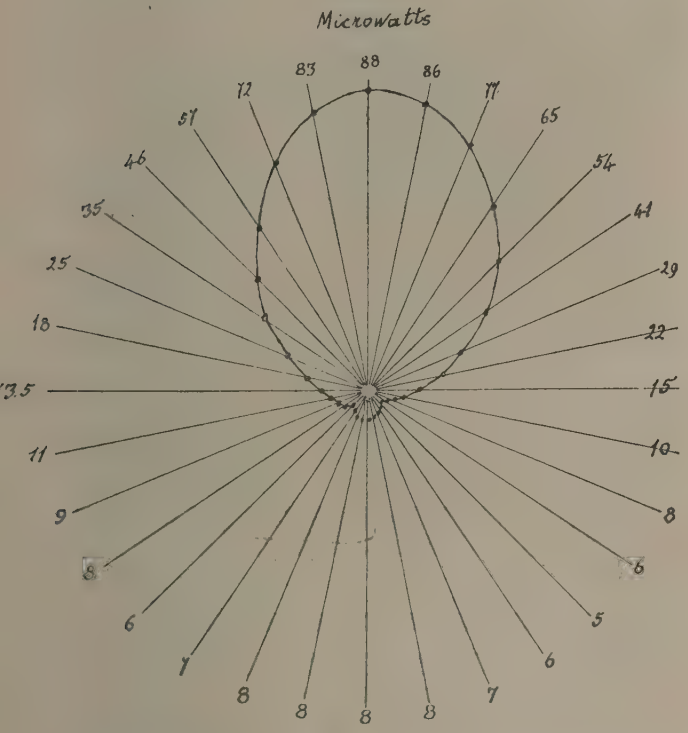
Fig. 11.



when using the arrangement shown in fig. 7 ; and that of fig. 12 with the arrangement of fig. 10. In the last diagram (fig. 12) the phase-difference between the radiation from the two component systems turns out to be much smaller than in the preceding diagram, and only by means of this last arrangement (fig. 10) have the very small values of the backwardly radiated energy been obtained. This proves the

existence of a phase-difference of 90°, or thereabouts, between the emissions from the two systems. The diagram of fig. 12 shows besides, that the problem of the emission of the energy exclusively on the useful side can be considered as having been practically solved.

Fig. 12.



It is interesting to note that, even if the phase-difference between the radiation from the two systems is as much as a certain number of degrees, and the ratio of the partial fields is slightly different from unity, the backward radiation is always practically zero. The phase-difference has to be over 45° for the backward radiation to become noticeable. In order to see if the energy emitted by each component of the

system and by the resultant system satisfied the theoretical predictions, these have been measured in several cases with the thermo-galvanometer. The results were always in practical agreement with the theory.

The simultaneous excitation of the two systems was effected by adding, internal to the primary of the transmitting radiogoniometer, a winding, fixed in position relatively to the primary and inductively excited by it. To this winding is connected the vertical antenna. On turning the movable winding of the apparatus the excitation of the vertical antenna remains constant whilst the excitations of the closed circuits forming the aerial of the bilateral system, vary. Consequently, to transmit by the unilateral system in a given direction, one has merely to turn the movable pointer until it points in this direction.

The tests of the unilateral system at long distance were made between the stations of Dieppe and Havre, and have confirmed the results obtained at short range—that is to say, on placing the pointer of the transmitting radiogoniometer in the direction of Havre, reception was effected ; on turning the pointer to 180° from this the reception ceased entirely.

Reception.—The same principle of the superposition of the two systems has been applied to the case of the reception. Considering the case of a receiving station provided with an aerial composed of a single closed oscillatory circuit, fixed in position, and of a vertical antenna symmetrically placed with respect to the closed oscillatory circuit, let it be supposed that any transmitting station is moving along a circumference of which the receiving station is the centre ; it is evident that the intensity and phase of the oscillatory current in the vertical antenna of the receiving station will remain constant, whilst in the closed oscillatory circuit they will vary with the position of the transmitting station. The intensity passes through two maxima and two minima, and its phase varies by 180° when the transmitting station passes from one side to the other of the plane perpendicular to the closed oscillatory circuit. Further, although a closed oscillatory circuit utilises the variation of magnetic flux across the surface bounded by such circuit, the E.M.F. generated in it will be in

quadrature with the E.M.F. generated in the vertical antenna, and the same phase-displacement will exist between the currents in the two types of aerials if these latter are both tuned to the length of the waves received.

Let it be supposed, for instance, that the closed oscillatory circuit is oriented in a certain meridian and that the transmitting station in moving about arrives in the plane of the closed oscillatory circuit, on the north side. In such a position, by bringing the actions of the two systems on the detector into phase by any means, the effects of the two will be added. When the transmitting station finds itself, similarly, in the plane of the closed circuit but on the south side, the effects of the two systems counteract one another. If the directive aerial of the receiving station is composed of two closed oscillatory circuits connected to the receiving radiogoniometer, the phase of the E.M.F. induced in the movable winding of the apparatus will vary by 180° when the coil is turned through 180°. So that the action of the directive aerial in one case will add itself to that of the vertical antenna, and in the other case will subtract itself therefrom. It follows that, to the position of maximum reception of the movable coil, there corresponds a position of zero or minimum reception when the coil is turned through 180°, which thus permits of the determination not merely of the direction of the transmitting station, but also of the side which it is on.

The action of the detector depends, as in the case of the transmission, upon the values of the phase-difference between the actions of the two systems and on the ratio M C. The law of variation of this action is the same as that of the variation of the resultant electromagnetic field at the transmitter and the diagrams are the same.

To obtain quadrature between the actions of the vertical antenna system and that of the directive bilateral system several arrangements have been tried which are shown in figs. 13, 14, 15.

In the case of fig. 13 the phase displacement was obtained by inserting an excess of self-inductance in one or other of the aerial circuits: in the arrangements of figs. 14 and 15

the shift of phase was produced automatically when the circuits were all carefully tuned, but the arrangement of fig. 14

Fig. 13.

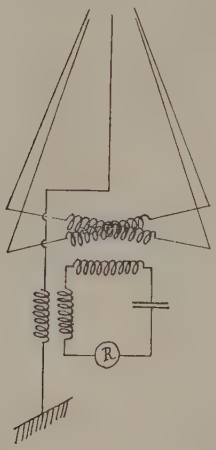


Fig. 14.

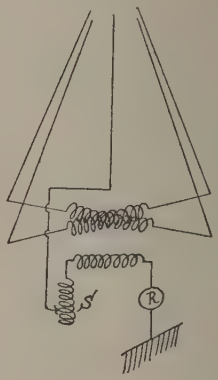
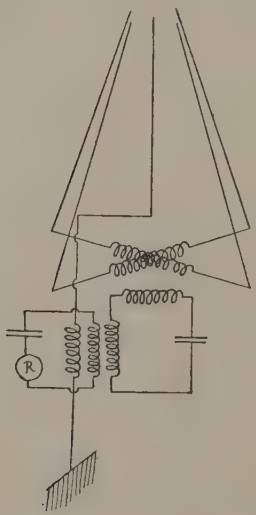


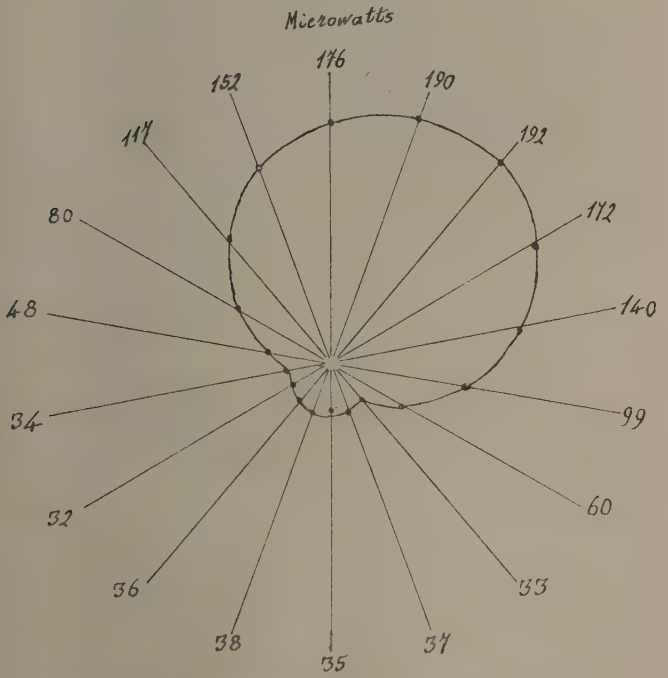
Fig. 15.



is preferable owing to its simplicity, rapidity, and certainty of operation. In this case, as can be seen from the figure,

the vertical antenna is joined to one end of the secondary circuit of the receiving radiogoniometer, through a self-inductance S ; the other end of the secondary is earthed through the detector R. The working of the arrangement is very simple, since it only requires the tuning of the two circuits as in ordinary cases of circular reception. The diagrams of the reception were obtained in the same way as

Fig. 16.



those of the transmission, by means of the thermo-galvanometer in place of the detector, and by turning the movable secondary of the receiving radiogoniometer, the transmitting station being fixed. The curve of fig. 16 in particular was obtained by means of the arrangement shown in fig. 13 ; the curve of fig. 17 (p. 324) by that of fig. 15 ; and the curve of fig. 18 (p. 324) by the arrangement of fig. 14.

Fig. 17.

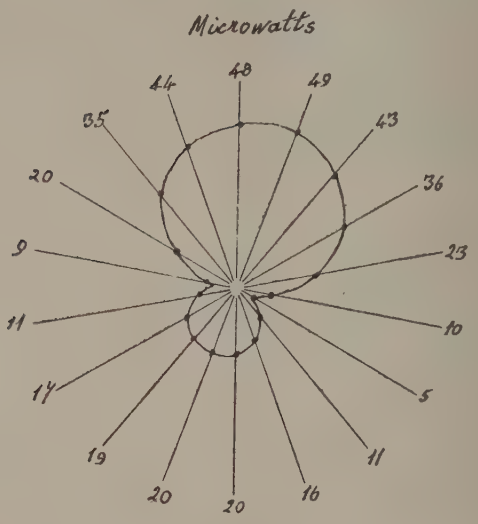

Microwatts
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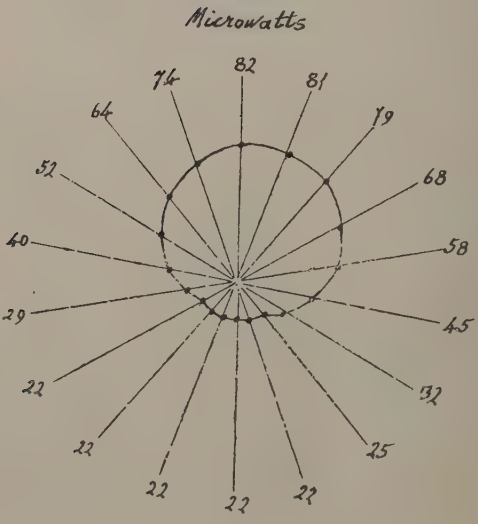
Fig. 18.


Microwatts
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The arrangements have always enabled the sense of the received transmissions to be determined, and in many cases to get no reception when the pointer of the movable coil of the receiving radiogoniometer was turned in the opposite direction, which was precisely the object to be achieved.

The directive system of wireless telegraphy described can be employed in many ways, according to the requirements, owing to the facility with which one can pass from one system to the other. For example, when the operator expects a message from a station whose position is unknown, he could employ the single vertical antenna system; a soon as reception is effected he can, by using either the bilateral or unilateral system, determine the direction and upon which side the transmitting station lies, and make himself independent of other transmissions.

For the transmission the operator could, similarly, employ the system with circular radiation when he does not know the position of the receiving station or when he wishes to send the same message to several stations. He could in other cases use the unilateral or bilateral system according to the position of the stations which should not receive the messages.

The directive system described presents some advantages over other systems, which are very important either from the commercial or the strategic point of view.

So far as the commercial services are concerned, the possibility of transmitting messages to one station without affecting the others, results in the stations having an increased working capacity and consequently in an enhancement of their commercial value. The ability to receive from one determined direction renders the receiving station independent of extraneous transmissions, and even owing to this circumstance alone the working capacity of the stations is raised. Further, the ability to determine the origin of a transmission, apart from its obvious advantages, presents the advantage of enabling the route to be followed by a ship proceeding to the assistance of another in danger, to be indicated.

From the strategic point of view the directive reception enables one to learn of the presence of the enemy in a certain direction and to follow him in his movements. The directive

transmission will allow, when suitably employed, of transmitting to one's friends without the enemy being able to receive the waves—for the simple reason that there are no waves there to be detected in the undesired region.

In conclusion, the following is given as one practical example of the main strategical advantage of the directive system. Supposing it were necessary to send radiotelegraphic messages to the cruiser squadrons on the north east, north, north-west, and west coasts of the United Kingdom; if the messages were sent by an ordinary vertical antenna system they would be perceived in Denmark, in Germany, in Belgium, Holland, and France. Employing, on the other hand, the unilateral system, the messages sent from London to the Fleet would not be received outside the United Kingdom.

The authors desire to thank Mr. L. H. Walter, M.A., for translating this paper from the original.

Discussion.

Dr Fleming said that he had long been acquainted with the interesting experiments of the Authors of this paper, and desired to express his congratulations on the skill and inventiveness with which these investigations had been prosecuted. He was glad to see that the experiments of the Authors confirmed in many ways the theory which he (Dr Fleming) had given of the operation of a bent antenna as employed by Mr Marconi.

When Mr Marconi read his paper at the Royal Society in March 1906 describing his experiments on directive telegraphy, he gave no theory in the matter, but Dr Larmor pointed out in the discussion that an antenna partly vertical and partly horizontal was equivalent to the sum of a magnetic oscillator and an electric oscillator, and shortly afterwards he (Dr Fleming) had gone more carefully into the matter mathematically and showed that the observed effects could be accounted for on this theory.

Both Mr Marconi and he himself had obtained by the same methods as those employed by Messrs. Bellini and Tosi the same type of pear-shaped radiation curves obtained by the Authors of the paper by combining together the effect of closed and open oscillators. Although this theory had been criticised by Dr Mandelstam lately, yet nevertheless there did not seem sufficient grounds for objecting to it. Mr Marconi had, as everyone knew, employed directive antennæ for a long time past in his Power Stations at Poldhu and Clifden, and had also given demonstrations showing that the position of ships out of sight could be located

by means of such receiving directive antennæ. Nevertheless, Messrs. Bellini and Tosi had worked out extremely ingenious arrangements for determining the direction of the radiant point without moving the antennæ themselves. He (Dr Fleming) had also shown that, having the power to locate the radiant point, two stations equipped with such antennæ at a known distance apart could by simultaneous observations determine also the distance of the radiant point, and this might become important in connection with marine work.

It ought also to be noticed that Dr F. Braun, of Strasburg, had by the employment of three open antennæ having oscillations in them of definite phase difference, been able to obtain radiation curves having the form of a cardioid similar to some of those given by the Authors of the paper.

Dr W. H. Eccles warmly congratulated the Authors upon their beautiful and original method. On this method, by merely rotating a small coil of wire on the table, a fixed aerial directive system of any size was made to do what could otherwise only be done by turning the whole system of aerial wires in azimuth. The essence of the system was the piece of apparatus styled the radiogoniometer, which, by causing appropriate component radiation from two fixed wire triangles set at right angles, brought about a resultant radiation in any direction desired—just as if a virtual aerial of the full size of the fixed aerials were being rotated in the air. Someone had compared the result so achieved with that obtained by Marconi's well-known arrangement of a number of fixed bent antennæ with their horizontal portions directed from a centre to various points of the compass. Apart from the fact that it is rarely feasible to fix up a large number of antennæ, the radiogoniometer method has the advantage that the direction of transmission or reception can be altered perfectly smoothly; so that if it can be used, as stated by the Authors, accurately to about one degree of arc, the radiogoniometer is the practical equivalent of 360 bent antennæ.

The looped aerials used by the Authors did not seem to the speaker to be the best kind of radiator for utilising the principle they had developed. The radiogoniometer would prove to be capable of giving excellent results with two fixed antennæ of the bent type set at right angles, one, for example, in the meridian plane and the other in the east-west plane. The two looped aerials of the Authors may each of them be assimilated to a pair of vertical antennæ emitting waves of 180° phase difference, and with the assumption that for a single vertical aerial the inverse square law holds for the propagation of electrical effects from a single aerial, the speaker showed by aid of the ordinary equation of wave-motion that a looped antenna obeying the condition stated emitted two waves of equal period, of phases differing 90° and 180° respectively from the phase of the radiation from either side of the loop, and of amplitudes having a ratio proportional to λ/x. This means that one portion of the resultant radiation obeys an inverse square law and the other portion an inverse cube law; the latter portion is the more important of the two when the distance x is small, the former is the more important when the distance x

is much greater than the wave-length λ. This explains why Dr Fleming's measurements of the radiation from closed oscillators, which were carried out at short distances, gave small promise of powerful propagation to a distance, and why Messrs Bellini and Tosi had found as a fact that good propagation occurred to great distances.

As regards the phases of the two portions of the radiation, the important portion for wireless telegraphy, namely that obeying the inverse square law, was 90° out of phase with the near side of the loop; and thus it is clear that the vertical aerial the Authors had used to cancel the radiation to one side of the sending station, must for that purpose emit radiation 90° out of phase with the near side of the loop. Messrs. Bellini and Tosi had reached this conclusion in the course of their experiments.

After eliciting that the power-supply to the primary of the induction-coil for the Dieppe-Havre experiments was 500 watts, Dr Eccles concluded by remarking that the Authors were worthy of special congratulation because in their method of unilateral transmission by phase and intensity adjustment of several radiators they had eluded the difficulties of managing the phases that had apparently baffled F. Braun when he was using the same principle.

Mr W. Duddell complimented the Authors upon their paper and asked for the wave-length of the waves given out by the transmitter. He also asked what would be the effect of opening the closed antennæ at the top.

Mr Tosi, in reply to Dr Fleming, stated that Braun's cardioid diagram was a theoretical diagram only; the method had not succeeded and no actual diagram obtained experimentally had ever been shown.

Referring to the similarity which Dr Fleming said the pear-shaped diagram of Marconi bore to the Authors' diagrams, Mr Tosi could only say that they had purposely shown some bad diagrams as well as some good ones. The good diagram of Marconi's corresponded to the Authors' bad diagram. With the Marconi horizontal aerial it would, moreover, be necessary, as Dr Eccles had pointed out, to employ 360 such wires to obtain an accuracy in locating the bearing of the radiant point equal to that secured with the system now described. Further, if the emitted energy is actually represented for longer distances by Marconi's pear-shaped diagram, it is difficult to understand why the transatlantic messages from Clifden to Glace Bay are readily picked up off the Algerian coast, at right angles to the line of transmission.

In reply to Mr Duddell he said that the wave-length of the waves employed was from 350 to 400 metres.

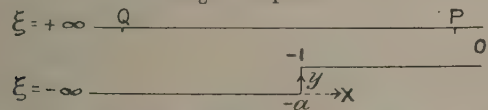
XXII. *On the Resistance of a Conductor of Uniform Thickness whose Breadth suddenly changes, and on the Shapes of the Stream-Lines in the immediate neighbourhood.* By CHARLES H. LEES, *D.Sc., F.R.S., Professor of Physics in the East London College, University of London* *.

A KNOWLEDGE of the resistance of a conductor whose section suddenly changes is of considerable practical importance, but mathematical difficulties have prevented an exact solution of the problem. Lord Rayleigh † has given an approximate solution of the case in which a cylinder of circular section is joined at one end to the plane surface of a large conducting solid, and Professor Hicks ‡ has solved the case of a wire of small diameter ending in the surface of a conducting sphere §.

The mathematical difficulties of the problem disappear if the conductors are of rectangular section and one dimension, *e. g.* the thickness, remains constant, while the other, the breadth, suddenly changes, and the two are joined together either with their axes or with two sides colinear. For a longitudinal flow through the conductors the former case becomes the latter by section through the common axis, which by symmetry is a line of flow.

Let the thickness of the conductor be t, the breadth of the wider part c, that of the narrower b, and let k be the conductivity of the material. Take the plane of the conductor

Fig. 1.—z plane.



as the z plane, fig. 1, and convert the outline of the conductor

* Read June 12, 1908.

† Lord Rayleigh, Trans. Roy. Soc. London, lxi. p. 77, Nov. 1870; and 'Scientific Papers,' vol. i. p. 56. See also Phil. Mag. viii. p. 481 (1904).

‡ Professor Hicks, Messenger of Mathematics, xii. p. 183 (1883).

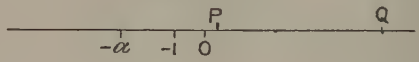
§ A list of cases which have been worked out will be found in Professor Auerbach's articles "Flachenströme" and "Körperliche Ströme" in Winkelmann's *Handbuch der Physik*, 2nd edit. iv. pp. 238, 246.

by means of the Schwarzian transformation

$$\frac{dz}{d\zeta} = A\zeta^{-1}(\zeta+1)^{\frac{1}{2}}(\zeta+a)^{-\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

into the axis of ξ in the ζ plane fig. 2, the corresponding

Fig. 2.—ζ plane.



points being indicated by the lettering in the figures.

In $$z = A\int \frac{1}{\zeta}\sqrt{\frac{\zeta+1}{\zeta+a}}\,d\zeta$$

write $$\frac{\zeta+1}{\zeta+a} = \frac{1}{\theta^2},$$

and the integral becomes

$$z = 2A\int \frac{(a-1)\,d\theta}{(a-\theta^2)(1-\theta^2)}$$

$$= 2A\int\left(\frac{1}{1-\theta^2} - \frac{1}{a-\theta^2}\right)d\theta$$

$$= 2A\left\{\arg\begin{pmatrix}\tanh\\ \coth\end{pmatrix}\pm\sqrt{\frac{\zeta+a}{\zeta+1}} - \frac{1}{\sqrt{a}}\arg\begin{pmatrix}\tanh\\ \coth\end{pmatrix}\pm\sqrt{\frac{\zeta+a}{a(\zeta+1)}}\right\};^*$$

where the origin in the z plane is taken at the point corresponding to $\xi=-a$, $\eta=0$ in the ζ plane, and tanh or coth is taken according to whether the quantity under the root sign is less than or greater than unity.

In order to show the connexion between the z and ζ planes graphically, we write

$$\zeta+a = r_a e^{i\theta_a}, \qquad \zeta+1 = r_1 e^{i\theta_1},$$

where r_a, θ_a, r_1, θ_1 are the coordinates of the point $\xi\eta$, with

* The form argtanh has been retained throughout because of the symmetry of the norm and amplitude when this is done.

respect to the points $-a$ and -1 (fig. 2), and

$$r_a^2 = r^2 + 2ar\cos\theta + a^2, \qquad \cot\theta_a = \frac{\cos\theta + \frac{a}{r}}{\sin\theta},$$

$$r_1^2 = r^2 + 2r\cos\theta + 1, \qquad \cot\theta_1 = \frac{\cos\theta + \frac{1}{r}}{\sin\theta}.$$

Resolving each term of the above equation into its norm and amplitude, we obtain

$$w = A\left[\operatorname{argtanh}\left(\pm\frac{\cos\frac{\theta_a-\theta_1}{2}}{\cosh\log\sqrt{\frac{r_a}{r_1}}}\right) - i\arctan\left(\pm\frac{\sin\frac{\theta_a-\theta_1}{2}}{\sinh\log\sqrt{\frac{r_a}{r_1}}}\right) + im\pi\right.$$

$$\left. -\frac{1}{\sqrt{a}}\left\{\operatorname{argtanh}\left(\pm\frac{\cos\frac{\theta_a-\theta_1}{2}}{\cosh\log\sqrt{\frac{r_a}{ar_1}}}\right) - i\arctan\left(\pm\frac{\sin\frac{\theta_a-\theta_1}{2}}{\sinh\log\sqrt{\frac{r_a}{ar_1}}}\right) + in\pi\right\}\right]$$

where m and n are integers. The alternatives tanh and coth of equation (2) both lead to the same result.

On determining the values of the constants A and a so as to make $y=0$ from $\xi=-\infty$ to $\xi=-a$, $y=c-b$ from $\xi=-1$ to $\xi=0$, $y=c$ from $\xi=0$ to $\xi=\infty$, we find that the negative sign must be assigned to the terms in the small brackets, that $A=c/\pi$, $\sqrt{a}=c/b$, and the equations become

$$z = x+iy = \frac{2b}{\pi}\arg\begin{pmatrix}\tanh\\ \coth\end{pmatrix}\frac{c}{b}\sqrt{\frac{\zeta+1}{\zeta+c^2/b^2}} - \frac{2c}{\pi}\arg\begin{pmatrix}\tanh\\ \coth\end{pmatrix}\sqrt{\frac{\zeta+1}{\zeta+c^2/b^2}}$$

$$= \frac{b}{\pi}\left\{\operatorname{argtanh}\frac{\cos\frac{\theta_1-\theta_a}{2}}{\cosh\log\frac{c}{b}\sqrt{\frac{r_1}{r_a}}} - i\arctan\frac{\sin\frac{\theta_1-\theta_a}{2}}{\sinh\log\frac{c}{b}\sqrt{\frac{r_1}{r_a}}}\right\}$$

$$-\frac{c}{\pi}\left\{\operatorname{argtanh}\frac{\cos\frac{\theta_1-\theta_a}{2}}{\cosh\log\sqrt{\frac{r_1}{r_a}}} - i\arctan\frac{\sin\frac{\theta_1-\theta_a}{2}}{\sinh\log\sqrt{\frac{r_1}{r_a}}}\right\},^{*} \quad (3)$$

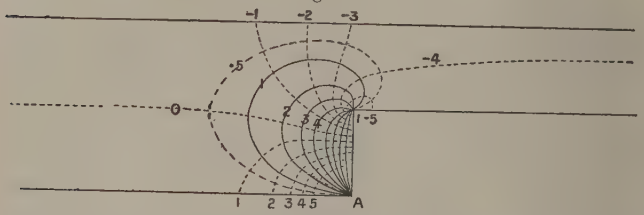
* The term $\operatorname{argtanh}\frac{\cos\theta}{\cosh\log\rho}$ may be more familiar in the forms

$$\tfrac{1}{2}\log\frac{\cosh\log\rho+\cos\theta}{\cosh\log\rho-\cos\theta} = \tfrac{1}{2}\log\frac{\rho^2+2\rho\cos\theta+1}{\rho^2-2\rho\cos\theta+1}.$$

where the arctan terms are to be taken within the limits 0 and π.

The curves in the z plane into which this equation converts the orthogonal system of circles $\theta_1 - \theta_a = \text{constant}$, and $r_1/r_a = \text{constant}$, of the ζ plane, are shown in fig. 3 for the

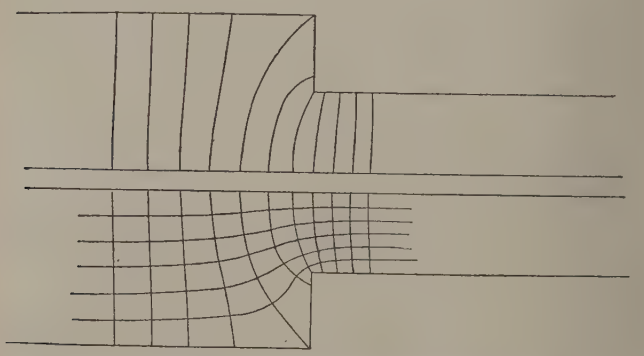
Fig. 3.



Full lines are equipotential lines, dotted lines are stream lines, when the end 1A is kept at potential 9, and the rest of the contour at 0. Ratio of breadths 1 : 2.

case $c/b = 2$. They may be considered either as the potential and stream curves for a source at 1 and an equal sink at A, or for the edge 1A kept at one potential and the whole of the remainder of the edges at another.

Fig. 4.



Lower part equipotential and stream lines calculated from (3); upper part equipotential lines observed. Ratio of breadths 1 : 2.

The curves in the z plane corresponding to $\log \zeta = \text{constant}$ in the ζ plane are shown in fig. 4 lower part, and represent

the equipotential and stream lines for a current flowing along the two strips in series. The upper part shows the observed equipotential lines *.

To determine the resistance of the compound conductor to the flow of current along its length, we find the distribution of the potential in the corresponding problem in the ζ plane. This is the case of a source at the point $\xi=\eta=0$, sending a current C into the upper half of the plane.

The vector function $w=u+iv$ for this case is given by the equation

$$w=\frac{C}{\pi kt}\log\zeta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where t is the thickness of the conductors and k is the conductivity of the material of which they are composed.

If $\xi_P, 0$, are the coordinates of a point P near O in the ζ plane, and x_P is its X coordinate in the z plane (figs. 1 and 2), then by equation (3)

$$x_P=\frac{2b}{\pi}\operatorname{argcoth}\frac{c}{b}\sqrt{\frac{\xi_P+1}{\xi_P+c^2/b^2}}-\frac{2c}{\pi}\operatorname{argtang}\sqrt{\frac{\xi_P+1}{\xi_P+c^2/b^2}},$$

which since ξ_P is small reduces to

$$x_P=\frac{2b}{\pi}\operatorname{argcoth}\sqrt{1+\frac{c^2-b^2}{c^2}\xi_P}-\frac{2c}{\pi}\operatorname{argtanh}\frac{b}{c}.$$

$$=\frac{b}{\pi}\left\{\log\frac{4c^2}{c^2-b^2}-\frac{\pi kt}{C}.u_P\right\}-\frac{c}{\pi}\log\frac{c+b}{c-b},$$

in virtue of equation (4).

Hence

$$\frac{\pi kt}{C}.u_P=-\frac{\pi}{b}x_P-\log\frac{c^2-b^2}{4c^2}-\frac{c}{b}\log\frac{c+b}{c-b}.$$

Similarly if $\xi_Q, 0$, are the coordinates of a point Q near $+\infty$ in the ζ plane, and x_Q is its X coordinate in the z plane,

* I have to thank my colleague Mr. W. H. White for making the observations on a thin strip of german silver transmitting an electric current, from which these curves have been drawn.

then by equation (3)

$$x_Q=\frac{2b}{\pi}\operatorname{argcoth}\frac{c}{b}\sqrt{\frac{\xi_Q+1}{\xi_Q+c^2/b^2}}-\frac{2c}{\pi}\operatorname{argtanh}\sqrt{\frac{\xi_Q+1}{\xi_Q+c^2/b^2}},$$

which, since ξ_Q is large, reduces to

$$x_Q=\frac{2b}{\pi}\operatorname{argcoth}\frac{c}{b}-\frac{2c}{\pi}\operatorname{argtanh}\sqrt{1-\frac{c^2-b^2}{b^2\xi_Q}}$$

$$=\frac{b}{\pi}\log\frac{c+b}{c-b}-\frac{c}{\pi}\left\{\log\frac{4b^2}{c^2-b^2}+\frac{\pi kt}{C}\cdot u_Q\right\}$$

in virtue of equation (4).

Hence

$$\frac{\pi kt}{C}u_Q=-\frac{\pi}{c}x_Q-\log\frac{4b^2}{c^2-b^2}+\frac{b}{c}\log\frac{c+b}{c-b}.$$

Thus

$$\frac{kt}{C}(u_Q-u_P)=\frac{x_P}{b}-\frac{x_Q}{c}+\frac{1}{\pi}\left(2\log\frac{c^2-b^2}{4cb}+\frac{c^2+b^2}{cb}\log\frac{c+b}{c-b}\right)$$

Hence the resistance between two transverse sections of the conductors through the points P and Q situated at considerable distances from the change of section and on opposite sides of it, is equal to the sum of the resistances of the lengths of conductor on the two sides of the change of section, each considered as part of an infinite length, plus the resistance of a length of either conductor equal to

$$\frac{1}{\pi}\left\{2\log_e\frac{c^2-b^2}{4cb}+\frac{c^2+b^2}{cb}\log_e\frac{c+b}{c-b}\right\}$$

times the breadth of that conductor.

When the ratio of the breadths $c/b=n$ becomes large, the correction approximates to

$$\frac{2}{\pi}(\log_e n-\cdot 386),$$

and when very large to

$$\frac{2}{\pi}\log_e n.$$

When, on the other hand, the ratio n of the breadths becomes nearly unity the correction approximates to

$$\frac{(n-1)^2}{\pi}\left\{\frac{1}{2}+\frac{1}{n}\log_\epsilon\frac{2}{n-1}\right\},$$

and when it is very nearly unity to

$$\frac{(n-1)^2}{\pi}\left\{1{\cdot}19-\log_\epsilon(n-1)\right\}.$$

In order to exhibit the character of this correction the following table has been calculated.

Correction to be added to the length of one conductor in terms of the breadth of that conductor, in order to make the total resistance equal to the sum of the resistances of the two conductors, each considered as part of a conductor of infinite length.

Ratio of breadths.	No. of breadths to be added to length.	Ratio of breadths.	No. of breadths to be added to length.
1·0	·0	3·0	·48
1·2	·032	4·0	·65
1·4	·080	6·0	·90
1·7	·17	10·0	1·24
2·0	·25	15·0	1·46
2·5	·37	20·0	1·65

Discussion.

Dr Russell congratulated the Author on having obtained the exact solution of an important problem and thanked him for giving it in a form in which it could be utilised readily by electricians. Somewhat similar problems are of frequent occurrence in practice, in particular he instanced the measurement of the resistance of the bonds connecting the rails in electric tramway systems. The difficulty in this case is in knowing where the rail ends and the bond begins.

A small variation in the position of the potential contacts makes a large variation in the reading of the galvanometer. In many cases the only way of attacking the problem is to calculate the resistance from the known resistivity of the metals by the approximate method indicated by Maxwell. An exact solution, therefore, like the one obtained by the Author, would be of great value in checking the accuracy of the approximate method.

FIG. 8.—SPECIAL CAST IRON NO. 2.

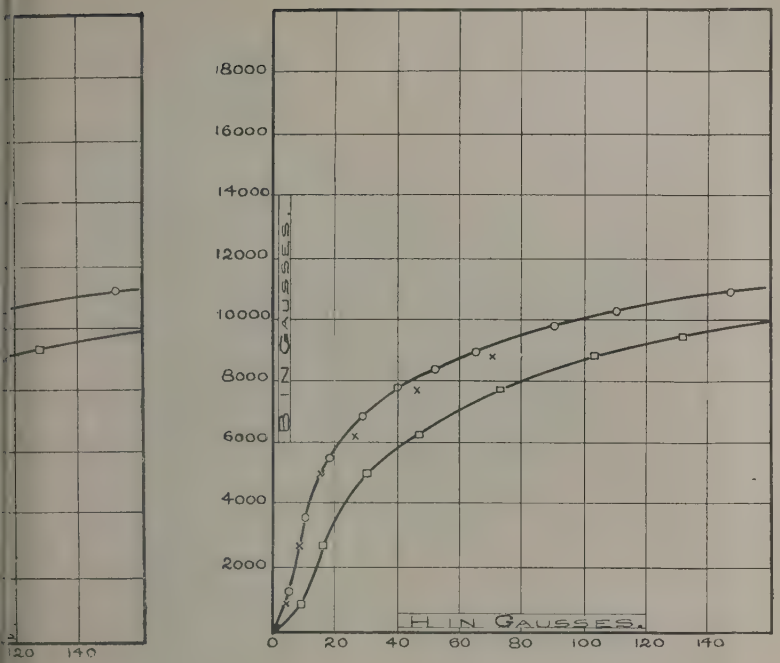


FIG. 9.—MILD STEEL NO. 2.

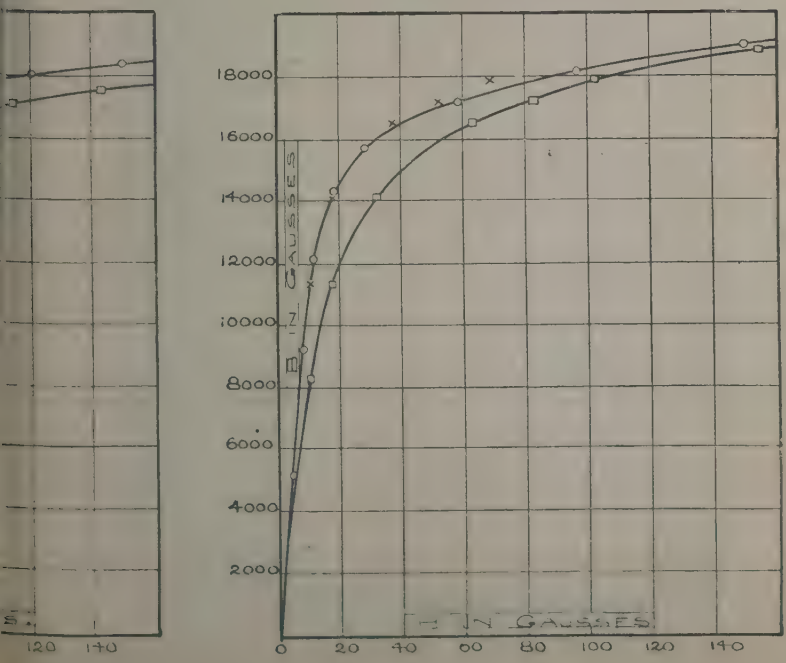


FIG. 12.

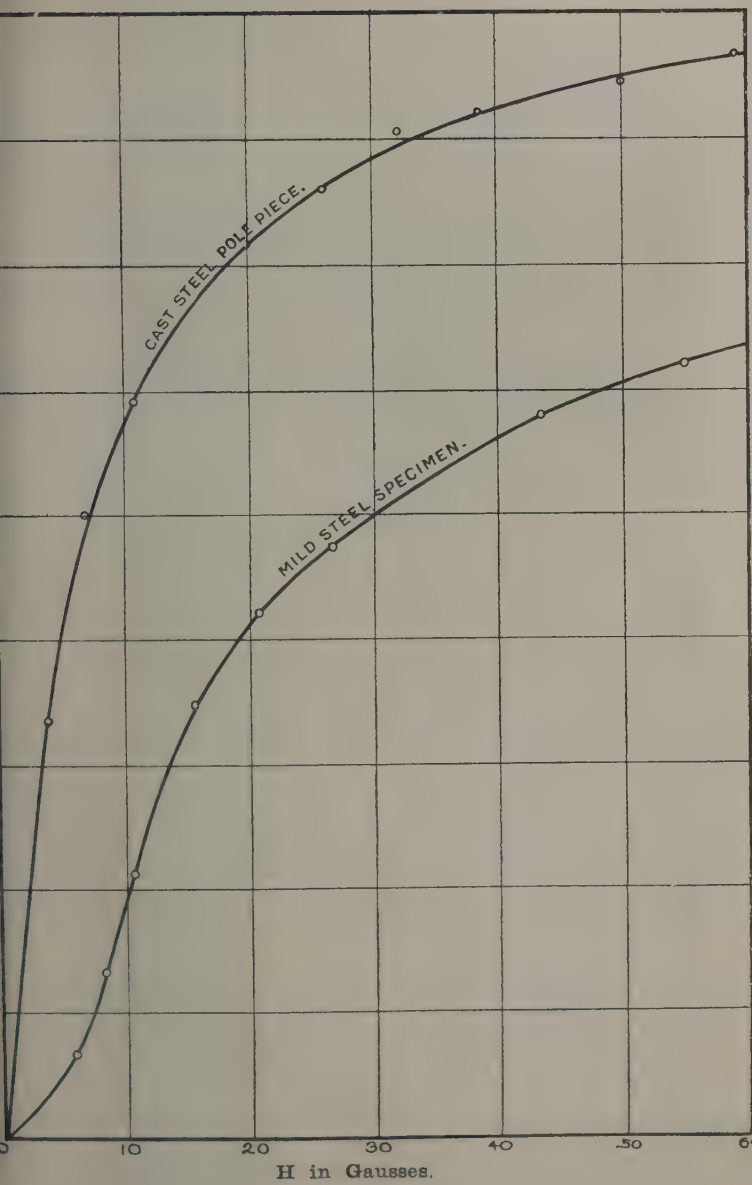

CAST STEEL POLE PIECE.
MILD STEEL SPECIMEN.
10
20
30
40
50
60
H in Gausses.

Fig. 7.—Ratio Tests on Current Transformer.

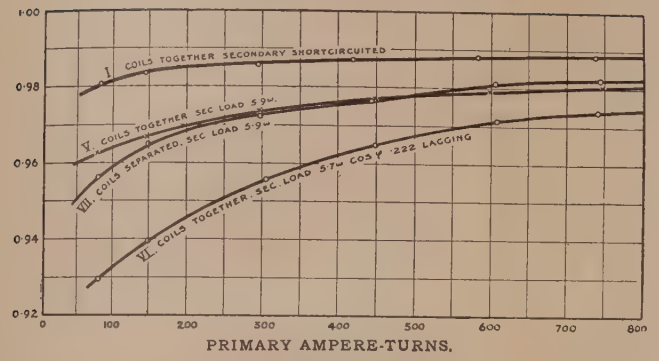


Fig. 8.—Phase Displacement on Current Transformer.

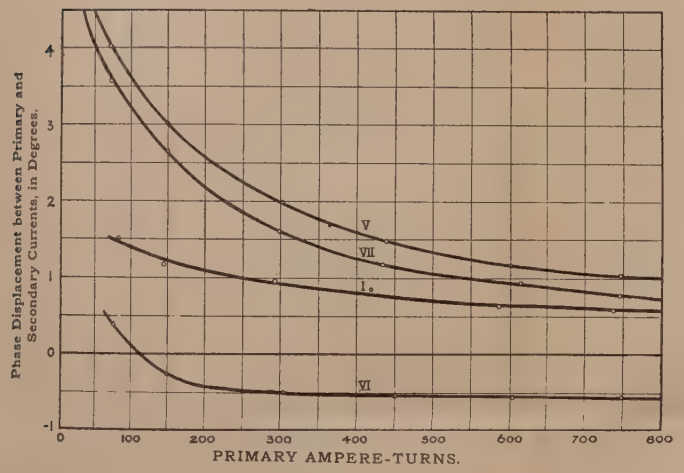


Fig. 9.—Ratio Tests on Voltage Transformer.

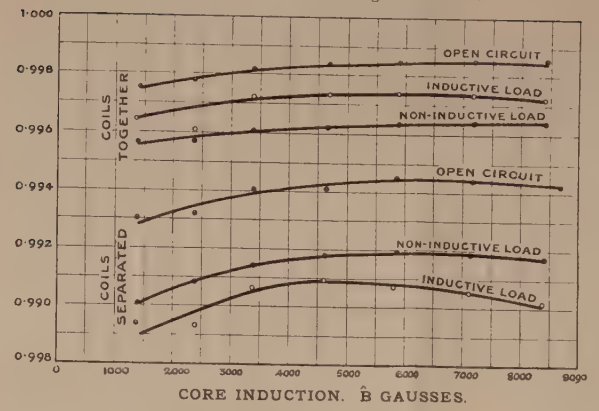


Fig. 10.—Displacement Tests on Voltage Transformer.

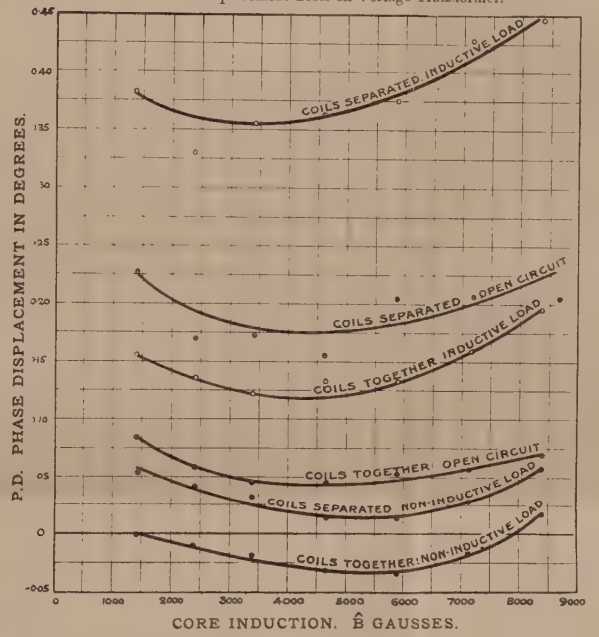


Fig. 11.—Relation between Core Loss and Magnetizing Current.

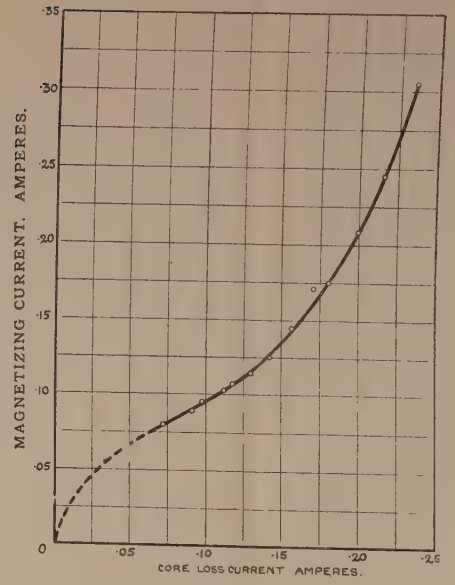


Fig. 12.—Relation between $\frac{A_c}{B}$ and $\frac{A_m}{B}$.

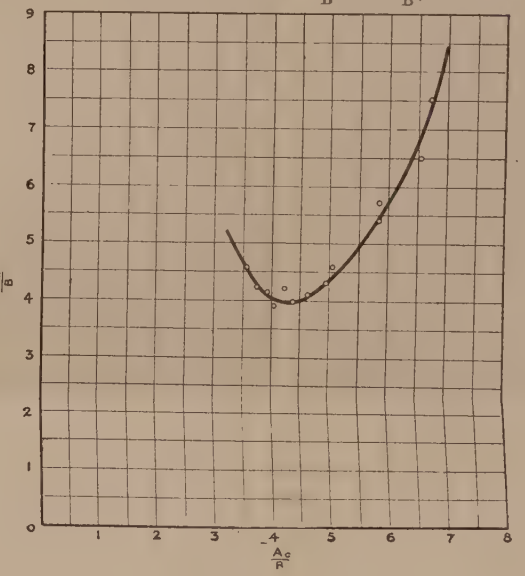


Fig. 13.—Ratio Tests on Current Transformers.

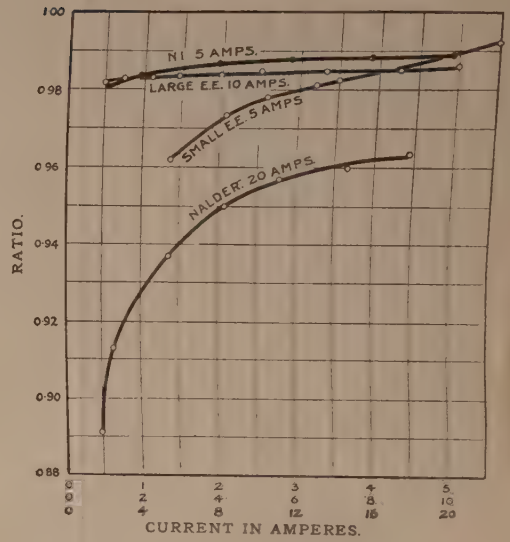
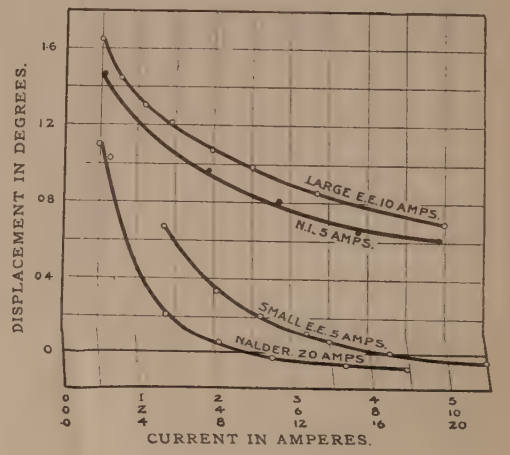


Fig. 14.—Displacement Tests on Current Transformers.



1. DIBLA. LAKE CHAD

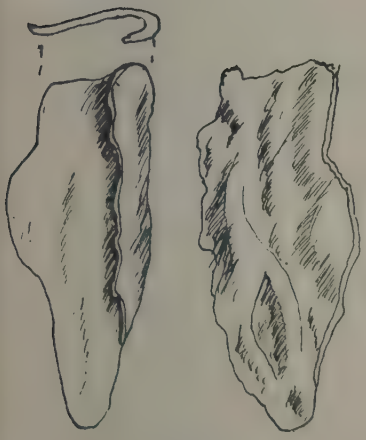
3. DRESDEN

2. ESSEX

FULGURITES.

4. DIBLA. LAKE CHAD

5. SENNER

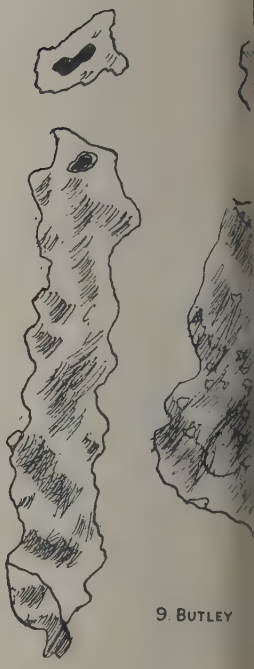


6. DRIGG

7. ESSEX

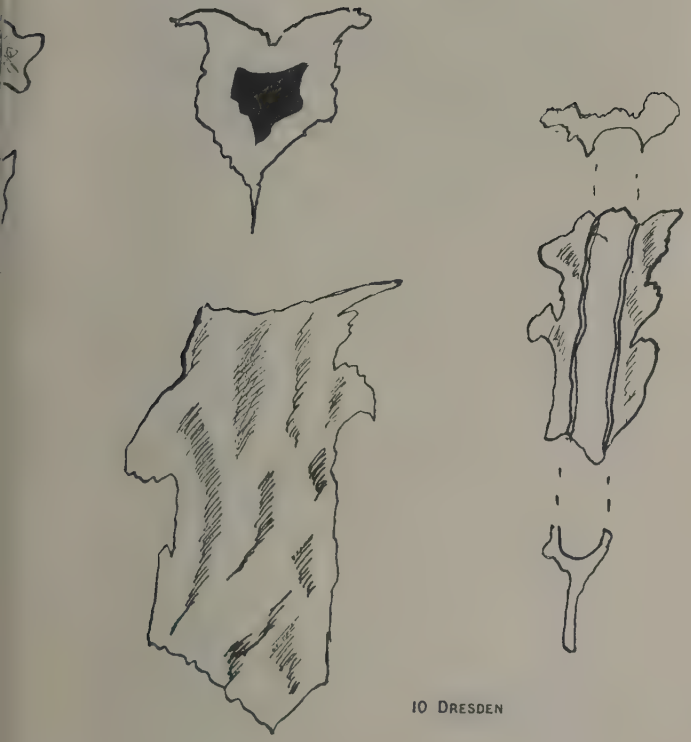
THICK-WALL

8. Senner

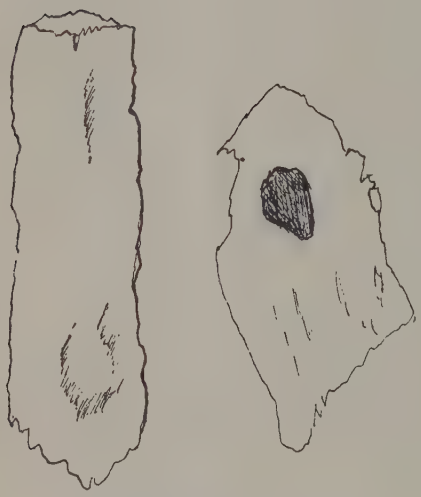


9. Butley

LGURITES.



10 DRESDEN

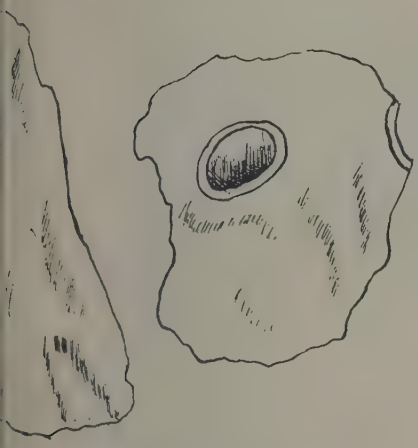


11. BORAX TUBES *(magnified)*
Actual size

13 TUBES OF CRYSTALLISED BORAX

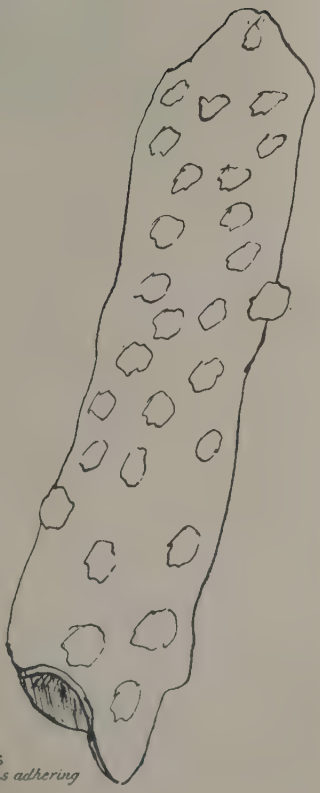
Actual size

L FULGURITES.



12. RESIN TUBES
encrusted in mass of powdered resin

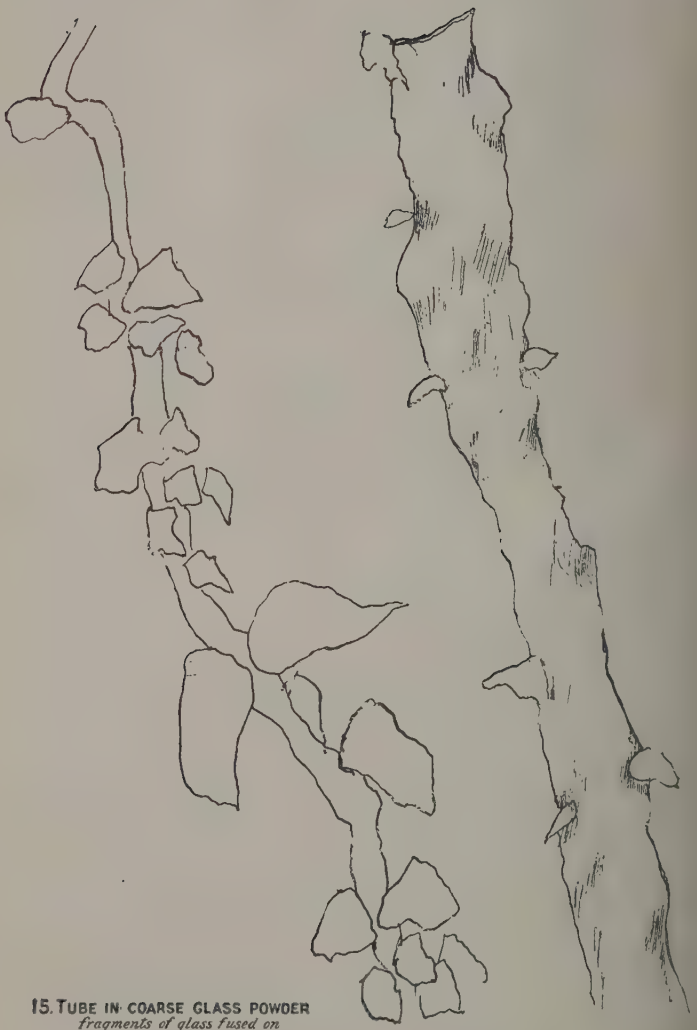
Actual size



14. SEALING WAX TUBES
with unmelted grains adhering

Actual size

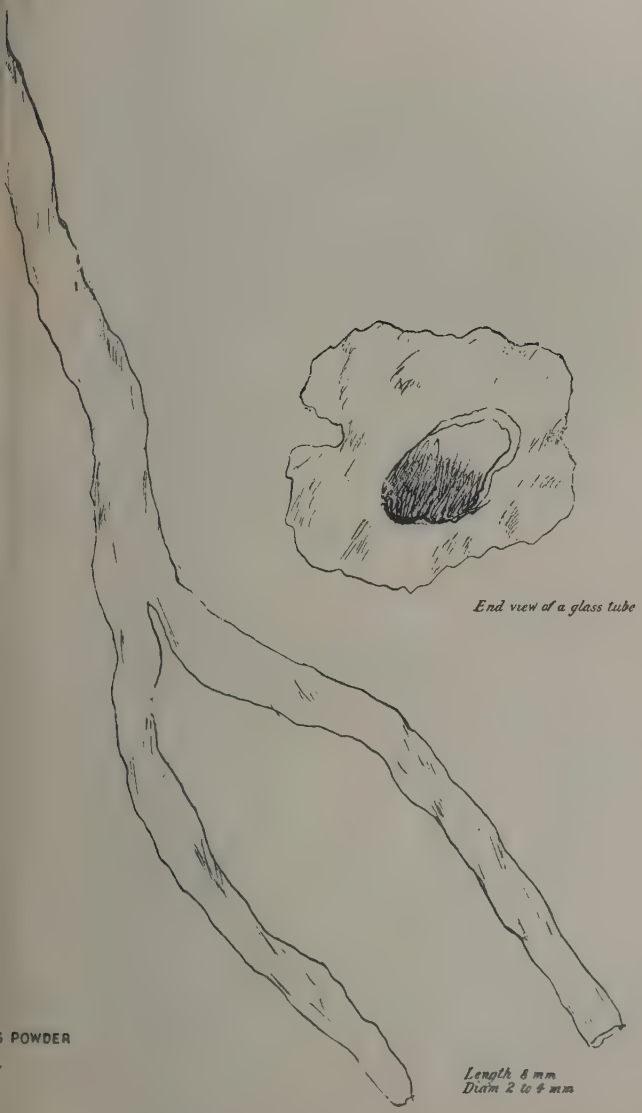
ARTIFICIAL FULG



15. TUBE IN COARSE GLASS POWDER
fragments of glass fused on
Actual size 6 mm long 2 mm diam

16 FI

TES IN GLASS.



End view of a glass tube

Length 6 mm
Diam 2 to 4 mm

S POWDER

P.D. and Current Wave-Forms of Short Sparks.

FIG. 3.—Spark-length equals zero.

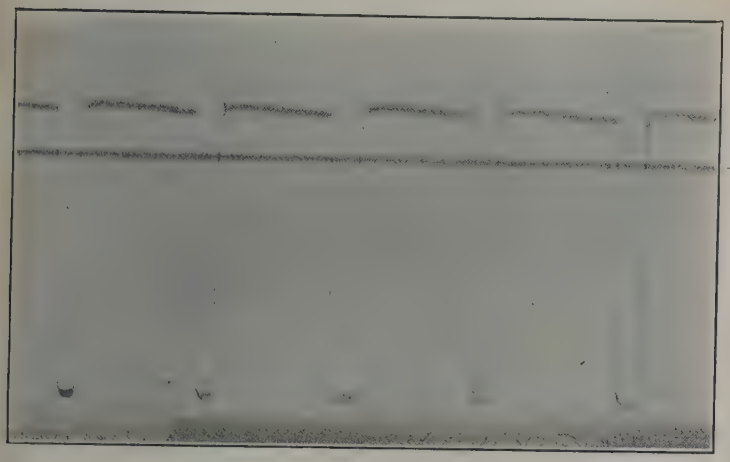
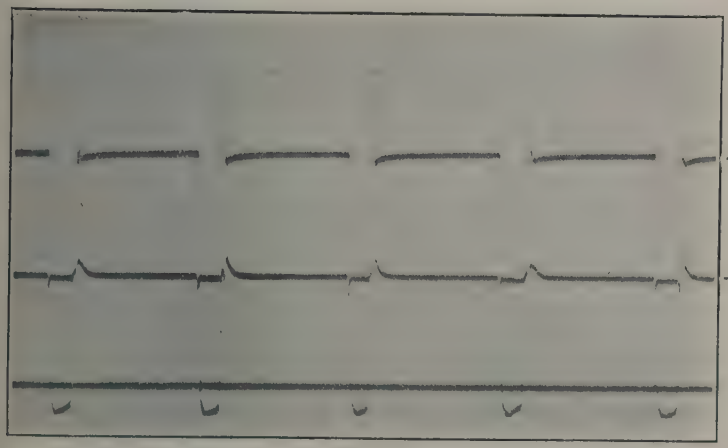


FIG. 4.—Spark-length 1 mm.



P.D. and Current Wave-Forms of Short Sparks.

FIG. 5.—Spark-length 5 mm.

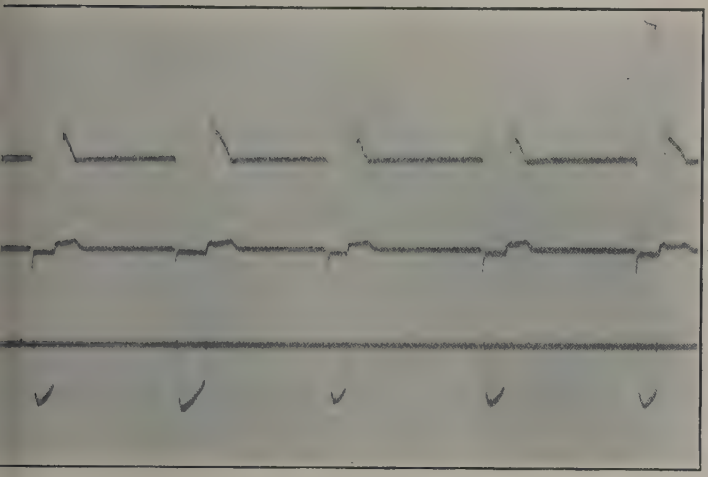
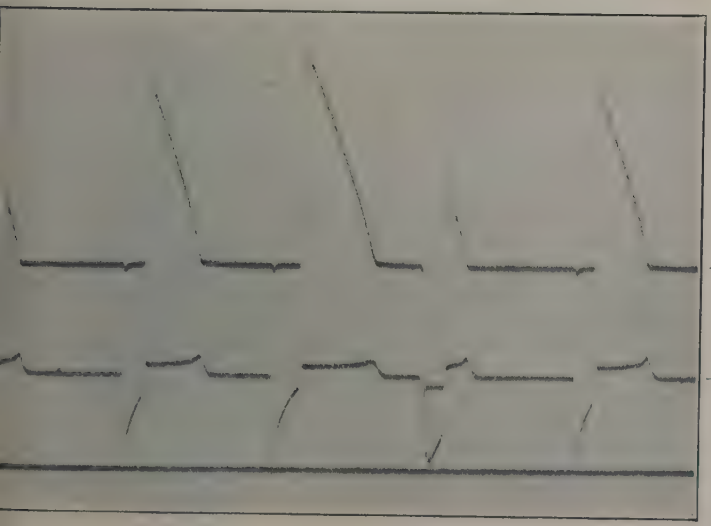
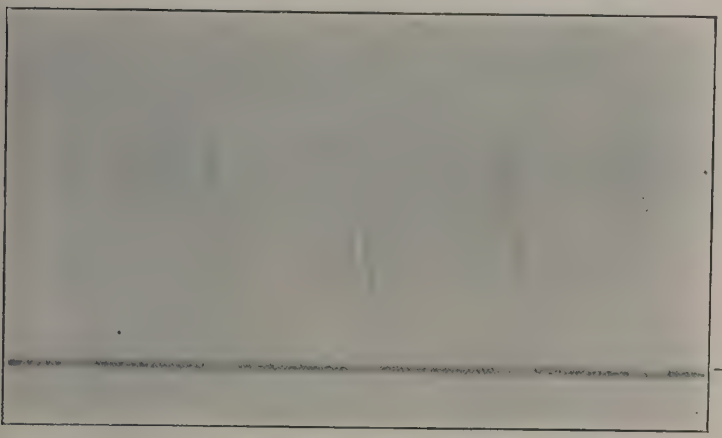


FIG. 6.—Spark-length 15 mm.



P.D. and Current Wave-Forms of Short Sparks.

FIG. 7.—Spark-length 30 mm.



P.D. Wave-Form not recorded.

FIG. 8.—Distance between electrodes too great for sparking to take place.

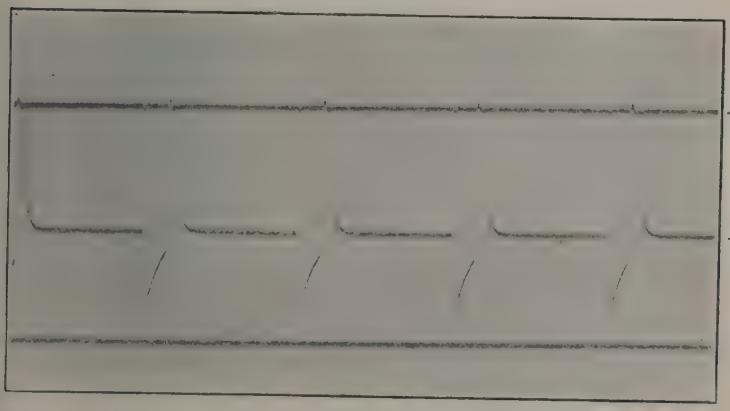
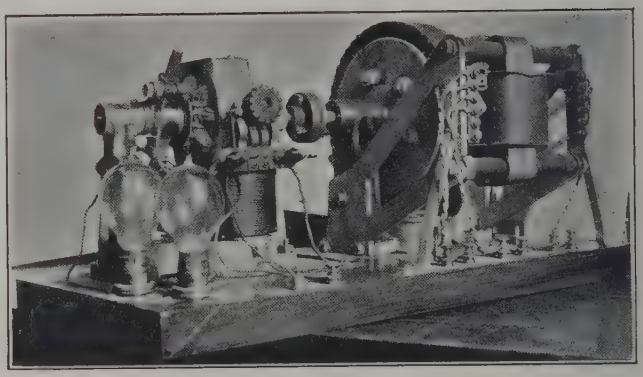
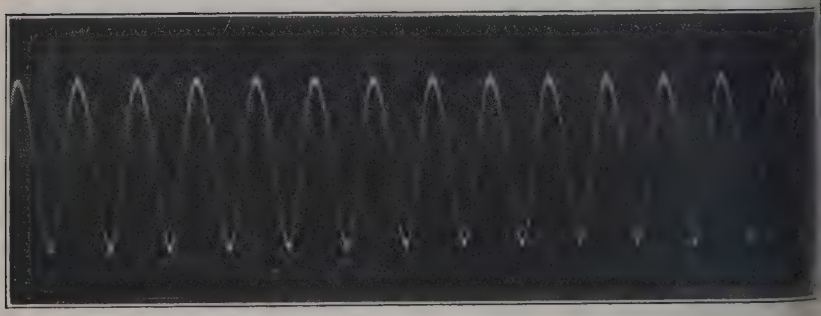


Fig. 1.



N. T. Co. Alternator.

Fig. 2.

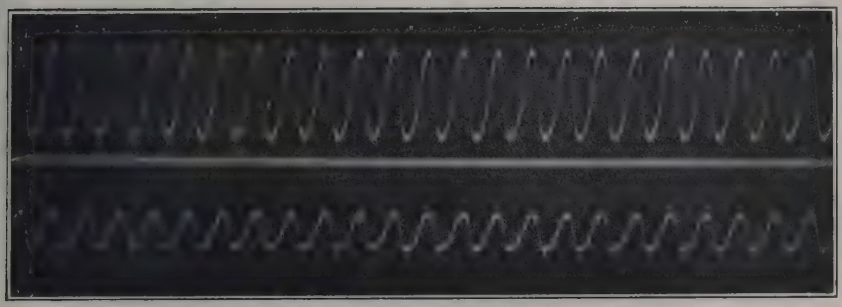


Unfiltered Wave.

Fig. 3.

Filtered Wave.

Fig. 4.



Waves in primary and secondary of humming telephone instrument.

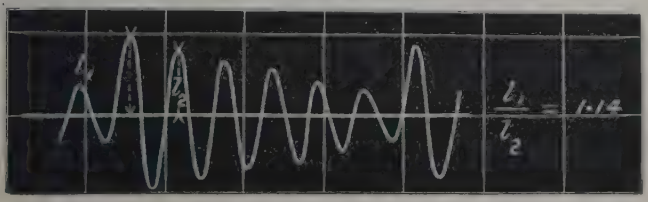
Fig. 18.

(*b*)

(*a*)

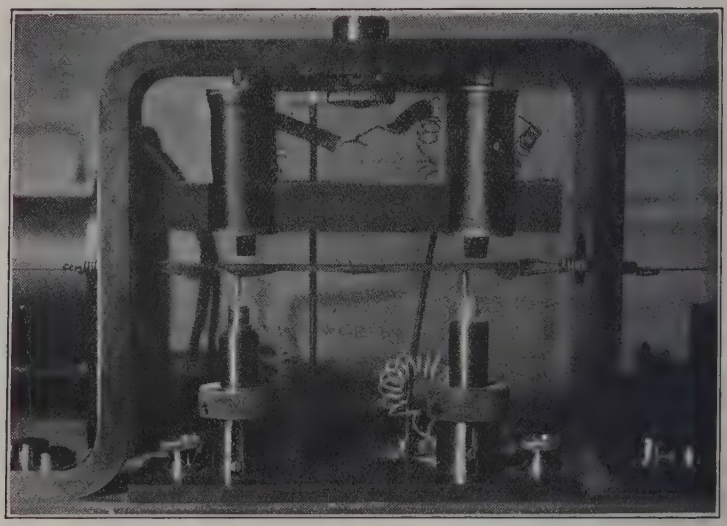
Wave Forms. Continuous oscillation circuit.

Fig. 19.



Mean Speech Wave.

FIG. 5.



VIBRATING WIRE.
Details of mercury cups, etc.

VIBRATING WIRE INTERRUPTER.
General View.

Fig. 7.

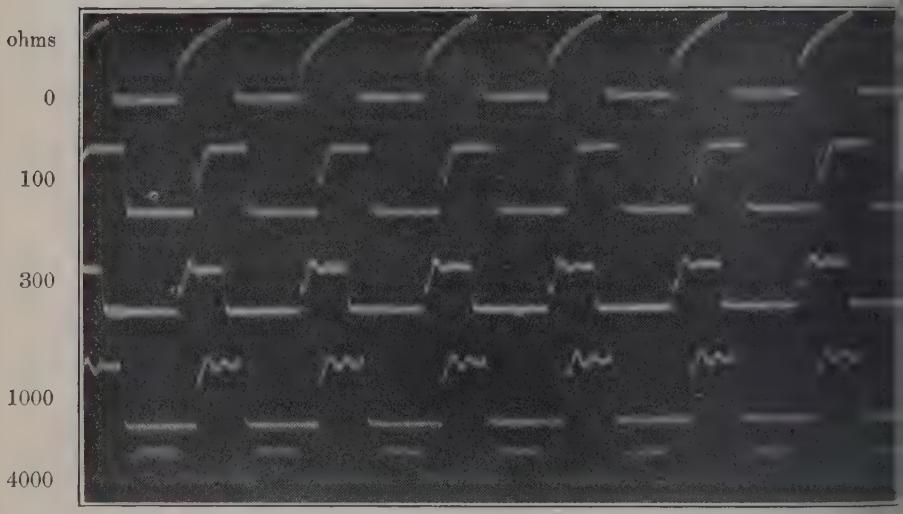
490 ~.

959 ~.

2880 ~.

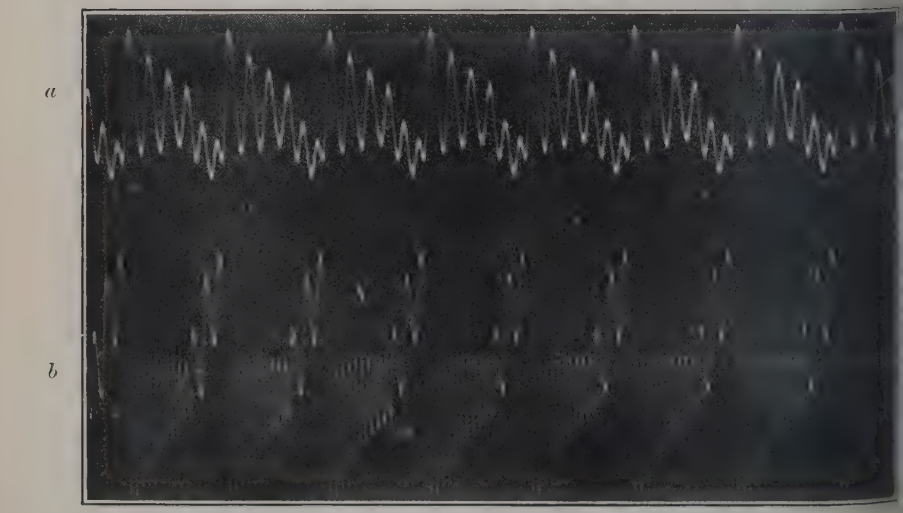
Single action circuit waves.

FIG. 9.



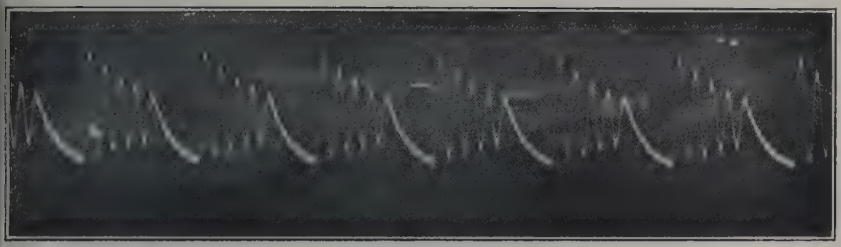
EFFECTS OF RESISTANCE IN SINGLE ACTION CIRCUIT.

FIG. 10.



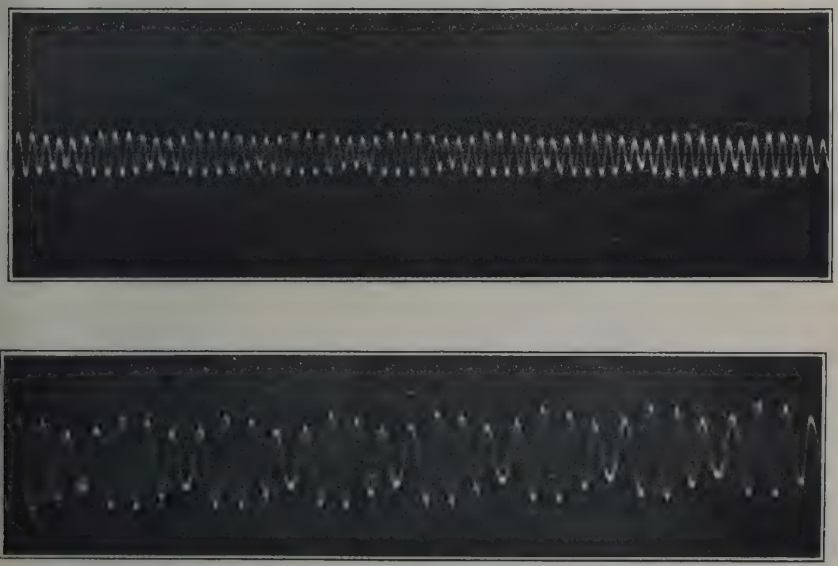
DOUBLE ACTION CIRCUIT WAVES.

Fig. 12.



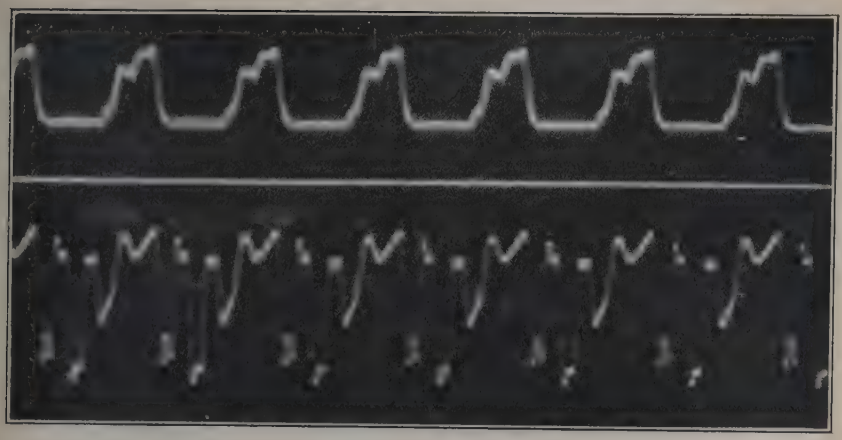
Assisting Waves.

Fig. 13.



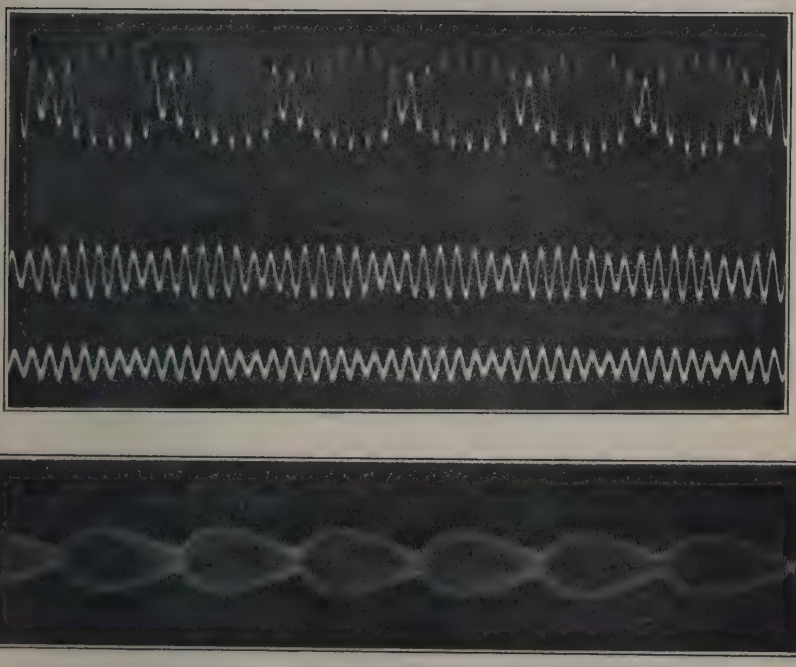
Opposing Waves.

FIG. 14.



COMPLEX WAVE FORM.

FIG. 16.



WAVE FORMS. Modified double action circuit.

FIG. 1.—General view of apparatus.

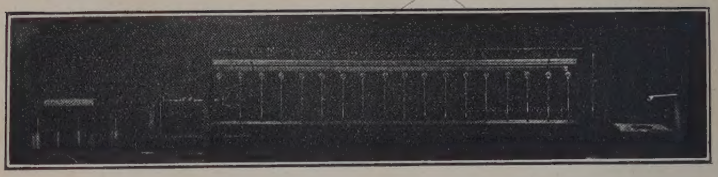


Fig. 2.—Apparatus and accessories.

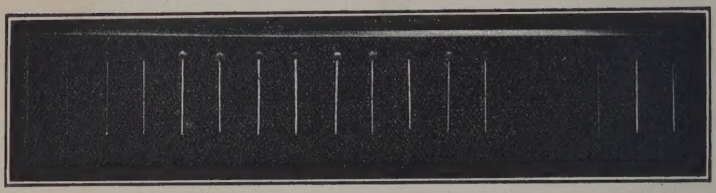


Fig. 3.—Frequency about 600,000. Receiving end open.

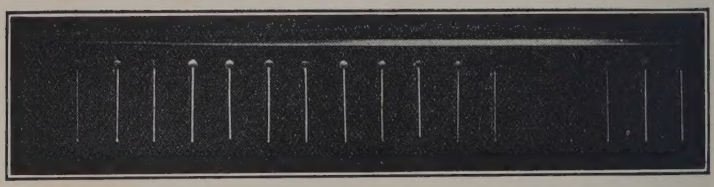


Fig. 4.—Frequency about 1,140,000. Receiving end open.

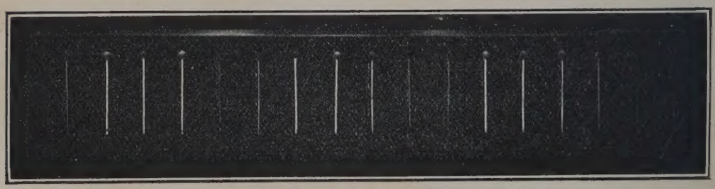


Fig. 5.—Frequency about 1,800,000. Receiving end open.

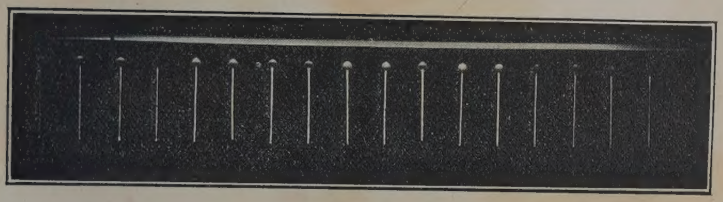


FIG. 6.—Frequency 600,000. Non-inductive receiver.

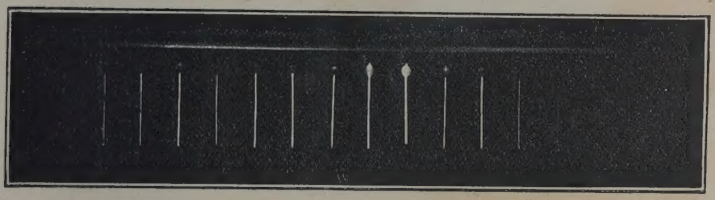


FIG. 7.—Frequency 600,000. Receiving end earthed.

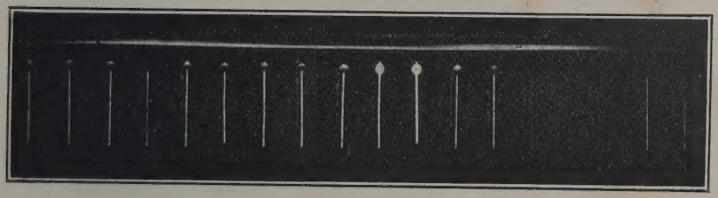


FIG. 8.—Frequency 600,000. Capacity increased by internal wire. Receiving end open.

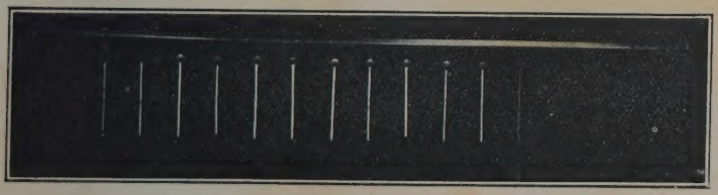


FIG. 9.—Frequency 600,000. Receiving end earthed through condenser.